CONSTRUCTAL DESIGN OF TWO T-SHAPED ASSEMBLIES OF FINS COOLING A CYLINDRICAL SOLID BODY

ROBERTA DE LIMA CORRÊA[1], LUIZ ALBERTO OLIVEIRA ROCHA[2], LIÉRCIO ANDRÉ ISOLDI[3], ELIZALDO DOMINGUES DOS SANTOS[4]

RESUMO

Este artigo considera o design constructal de um corpo cilíndrico sólido arrefecido por duas aletas na forma T. O objetivo é minimizar o máximo excesso de temperatura entre o corpo cilíndrico e sólido a temperatura ambiente. A geração interna de calor é distribuída uniformemente em todo o corpo sólido. As aletas de são banhadas por uma corrente constante, com temperatura ambiente constante e transferência de calor por convecção. As superfícies exteriores do corpo cilíndrico são adiabáticas. O volume total do corpo e o volume total do conjunto de aletas são fixos, mas a espessura e comprimento das aletas podem variar. A geometria otimizada e o desempenho são reportados graficamente como funções da relação entre a espessura e a razão entre os comprimentos das aletas. É também digno de que, quando comparada com uma configuração com um único conjunto de aletas na forma de T a configuração com duas aletas na forma de T realizam aproximadamente 50% melhor o seu desempenho.

PALAVRAS-CHAVE: Design Constructal. Geração de calor. Corpo cilíndrico sólido. Aletas em forma de T.

CONSTRUCTAL DESIGN OF TWO T-SHAPED ASSEMBLIES OF FINS COOLING A CYLINDRICAL SOLID BODY

ABSTRACT

This paper considers the constructal design of the architecture of two T-shaped assemblies of fins cooling a cylindrical solid body. The objective is to minimize the maximal excess of temperature between the solid cylindrical body and the ambient. Internal heat generating is distributed uniformly throughout the solid body. The assemblies of fins are bathed by a steady stream with constant ambient temperature and convective heat transfer. The outer surfaces of the cylindrical body are adiabatic. The total volume of the body and the total volume of the assembly of fins are fixed, but the thickness and lengths of the fins can vary.

[1]Doutoranda em Engenharia Mecânica – Universidade Federal do Rio Grande do Sul – UFGRS. Email: robertalimacorre@gmail.com
[2]Professor Adjunto da Escola de Engenharia - Universidade Federal do Rio Grande do Sul – UFRGS. Email: laorocha@gmail.com
[3] Professor Adjunto da Escola de Engenharia – Universidade Federal do Rio Grande – FURG. Email: liercioisoldi@gmail.com
[4]Professor Adjunto da Escola de Engenharia – Universidade Federal do Rio Grande – FURG. Email: elizaldodossantos@gmail.com

The optimized geometry and performance are reported graphically as functions of the ratio between the thicknesses and the ratio between the lengths of the fins. It is also to worth that when compared with a configuration with only one T-shaped assembly of fins the configuration with two T-shaped assembly of fins perform approximately 50% better.

KEY-WORDS: Constructal design. Heat generation. Cylindrical solid body. T-shaped assembly fins.

1. INTRODUCTION

Increasing numbers of people have been applying Constructal theory [1-5] to optimize the performance of flow systems by generating geometry, flow structure and to explain natural self-organization. Recent references [6,7] demonstrate that the most basic features of tree and forest architecture can be put on a unifying theoretical basis provided by the Constructal Law. Trees and forests are studied as integral components (along with river basins, atmospheric and oceanic circulation, etc) of the much greater global architecture that facilitates the cylindrical flow of water in nature and the flow of stresses between wind and ground.

The same principle employed for the determination of natural configuration across the board (river basins, turbulence, animal design, vascularization, locomotion, cracks in solids, dentritic solidification, earth climate, droplet impact configuration, etc) is also used to yields new designs for electronics, fuel cells, and tree networks for transport of people, goods and information [4,8,9]. The maximization of the rate of heat transfer in a given volume has become the basic principle of designing structures for heat and fluid flow. An important thermal design constraint is that temperatures have not to exceed a certain threshold. This approach is consistent to the constructal method in its original focus [1-3], where design is the result of a “permanent struggle for better and better global system performance under global constraints”.

This paper documents numerically the relation between the maximization of global performance and the morphing architecture of a flow system. The present numerical study aims to discover, by means of the Constructal Method, the optimal geometric configuration of two T-shaped assemblies of fins cooling a cylindrical solid body, with uniform internal heat generation. The assemblies of fins are bathed by a steady stream with constants ambient temperature and convective heat transfer, while the solid body has adiabatic conditions on the outer surface.

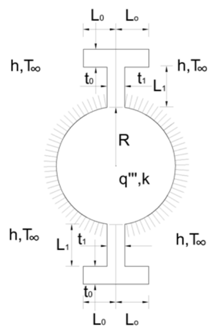
The total volume of the body and the total volume of the fins are fixed, but the fins lengths are free to vary. The optimized geometry and performance are reported graphically as function of the ratio between the thicknesses and the ratio between the lengths of the fins. Volume-to-point (or area-to-point) heat conduction problem was initially defined in

References [1,2] as follows: “consider a finite volume heated uniformly, with a finite amount of high conductivity material. Determine its optimal distribution through the given volume such that the highest temperature, i.e. the hot spot, is minimized.”

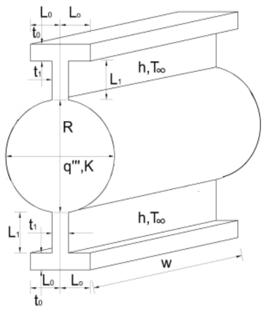
The problem statement here treated is not conceptually dissimilar from the above mentioned Bejan’s conduction problem: in this paper we attach fins to remove heat by means of the convection mechanism instead of the insertion of the high conductivity material, i.e. the conduction pathway.

2. MATERIAL AND METHODS

Consider the domain shown in FIGURE 1. There is an adiabatic cylindrical body with internal constant heat generation per unit of volume q’’’ [W/m^3] and constant thermal conductivity k [W/m.K]. Attached to the cylinder are two T-shaped assembly of fins. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with the diameter of the cylinder. The heat transfer coefficient *h* is uniform over all the exposed surfaces of the T-shaped assembly of fins and the temperature of the fluid (T_∞) is known. The maximum temperature (T_{max}) occurs into the cylinder and varies with the geometry of the two T-shaped assembly of fins.



(a)



(b)

Figure 1 - Two T-shaped assembly of fins cooling a cylindrical solid body.

The objective of the analysis is to determine the optimal geometry (L_1/L_0, t_1/t_0) that is characterized by the maximal excess of temperature $(T_{max} - T_\infty)/q'''A/k_b$, where k_b is the thermal conductivity of the body. According to constructal design [5], this optimization can be subjected to constraints, namely, volume of the cylindrical body constraint,

$$A = \pi R^2 \quad (1)$$

the fin-material area constraint,

$$A_f = N(2L_0t_0 + L_1t_1) \quad (2)$$

where N is defined as the number of T-shaped assemblies of fins, and the approximate area occupied by one fin (which is defined with the purpose to reduce one degree of freedom of the problem, decreasing the complexity of the optimization process),

$$Ac = 2L_0L_1 \quad (3)$$

Equations (2) and (3) can be expressed as the fin area fraction,

$$\phi = \frac{A_f}{A} \quad (4)$$

and the fraction of the area occupied by one fin,

$$\psi = \frac{A_c}{A} \quad (5)$$

The analysis that delivers the maximal excess of temperature as a function of the assembly geometry consists to solve numerically the heat conduction equation along the entire domain. The solid body is governed by the steady heat conduction equation with heat generation

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \quad (6)$$

while the heat conduction equation without heat generation is applied in the two T-assembly of fins

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \quad (7)$$

where the dimensionless variables are

$$\theta = \frac{T - T_\infty}{q''' A \big/ k_b} \quad (8)$$

and

$$\tilde{x}, \tilde{y}, \tilde{L}_0, \tilde{L}_1, \tilde{t}_0, \tilde{t}_1, \tilde{R} = \frac{x, y, L_0, L_1, t_0, t_1, R}{A^{1/2}} \quad (9)$$

The outer surfaces of the cylindrical solid body are insulated and the boundary conditions are given by

$$\frac{\partial \theta}{\partial \tilde{n}} = 0 \quad (10)$$

while the boundary conditions on the fin surfaces are given by

$$-\frac{\partial \theta}{\partial \tilde{y}} = \lambda \theta \qquad \text{or} \qquad -\frac{\partial \theta}{\partial \tilde{x}} = \lambda \theta \quad (11)$$

where the parameter λ is defined by [11]

$$\lambda = \frac{h A^{1/2}}{k_f} \quad (12)$$

and k_f is the thermal conductivity of the fin.

The dimensionless form of Equations (1) to (5) are

$$1 = \pi \tilde{R}^2 \quad (13)$$

$$\phi = N\left(2\tilde{L}_0\tilde{t}_0 + \tilde{L}_1\tilde{t}_1\right) \quad (14)$$

$$\psi = 2\tilde{L}_0\tilde{L}_1 \quad (15)$$

The maximal excess of temperature, θ_{max} , according to Equation (8) is given by

$$\theta_{max} = \frac{T - T_\infty}{\dfrac{q''' A^{1/2}}{k_b}} \quad (16)$$

3. RESULTS

It is not possible to express the global objective function in analytical form, in terms of the geometric parameters of the solid cylindrical body and the T-shaped assembly of fins. This function can be determined numerically, by solving for the temperature field in every assumed configuration to see whether the excess of temperature can be minimized by varying the configuration.

Equations (6) and (7) were solved using a finite elements code, based on triangular elements, developed in MATLAB environment, precisely the PDE (partial-differential-equations) toolbox [12]. Details about the optimization method, numerical method, mesh refinement and validation can be found in [11].

The structure shown in FIGURE 1 has two degrees of freedom: L_1/L_0, t_1/t_0. We start the simulations varying the ratio t_1/t_0 and keeping constant the other degree of freedom. FIGURE 2 shows that there is an optimal ratio $(t_1/t_0)_o$ that minimizes the maximal excess of temperature, θ_{max}, when the ratio L_1/L_0 is fixed as well the parameters ϕ, ψ, and λ. It is a shallow minimum for small values of the ratio L_1/L_0 and becomes more pronounced when the ratio L_1/L_0 increases.

The results of FIGURE 2 were summarized in FIGURE 3, which presents the once minimized maximal excess of temperature, $(\theta_{max})_m$, and the once optimized ratio $(t_1/t_0)_o$, as function of the ratio L_1/L_0. This Figure indicates for the case when the number of T-shaped fins is N = 2 that $(\theta_{max})_m$ decreases approximately 82% and $(t_1/t_0)_o$ increases dramatically when the ratio L_1/L_0 decreases from 1 to 0.01. FIGURE 3 also shows the behavior of the configuration with only one T-shaped assembly of fins, N =1. The configuration with two T-shaped assemblies of fins performs approximately 50 % better than the configuration with only one T-shaped assembly of fins. The best shapes and performance that emerged in FIGURE 3 are shown in scale in FIGURE 4.

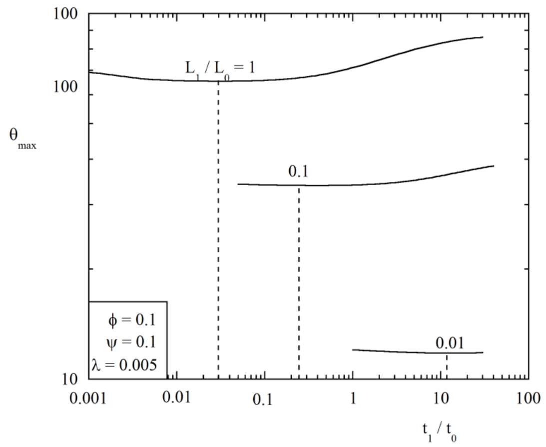


Figure 2 – The optimization of the maximal dimensionless excess of temperature as a function of the ratio t_1/t_0 for several values of the ratio L_1/L_0.

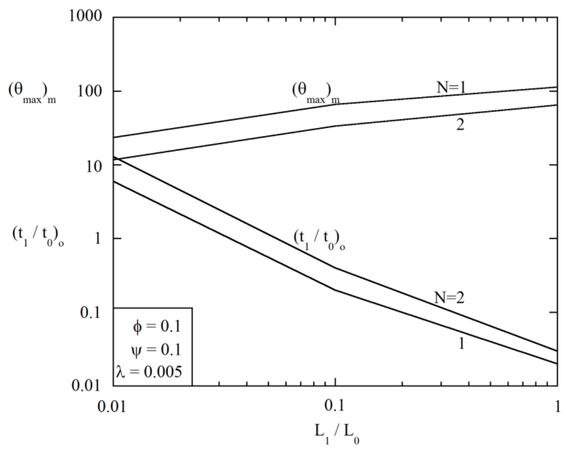


Figure 3 - The optimization of the once minimized dimensionless maximal excess of temperature and the corresponding once minimized ratio $(t_1/t_0)_o$ as a function of the ratio L_1/L_0.

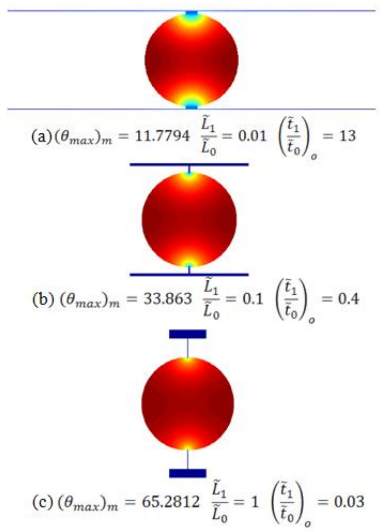


Figure 4 - The best shapes calculated in Figure 3.

4. CONCLUSIONS

This work applies constructal method to obtain the optimal geometric architecture of two T-shaped assemblies of fins cooling a cylindrical solid body, with uniform internal heat generation. The assemblies of fins are bathed by a steady stream with constant ambient temperature and convective heat transfer, while the solid body has adiabatic conditions on the outer surface.

The results indicate that there is an optimal ratio $(t_1/t_0)_o$ that minimizes the maximal excess of temperature, $(\theta_{max})_m$ when the ratio L_1/L_0 is fixed as well the parameters ϕ, ψ, and λ. It is also important to notice that the once minimized maximal excess of temperature, $(\theta max)m$,, decreases approximately 80% and the optimal ratio $(t_1/t_0)_o$ increases significantly as the ratio L_1/L_0 decreases from 1 to 0.01. When compared to the configuration with only one T-shaped assembly of the fins the configuration with N = 2 performs approximately 50% better.

REFERENCES

[1] BEJAN A. **Advanced Engineering Thermodynamics**, second ed., New York, USA. Wiley. 1997. Chapter 13.

[2] BEJAN, A. Constructal-theory network of conducting paths for cooling heat generating volume. **International Journal of Heat and Mass Transfer**, v. 40, p. 799-816, 1997.

[3] BEJAN, A. **Shape and Structure, from Engineering to Nature**, , Cambridge, UK. Cambridge University Press, 2000.

[4] BEJAN, A.; LORENTE S. Constructal theory of generation of configuration in nature and engineering. **Journal of Applied Physics**, v.100, p. 041301, 2006.

[5] BEJAN, A.; LORENTE S. **Design with Constructal Theory**, Hoboken Wiley, 2008.

[6] BEJAN, A.; LORENTE, S.; LEE, J. Unifying constructal theory of tree roots, canopies and forests. **Journal of. Theoretical Biology**, v. 254, p. 529-540 , 2008

[7] BEJAN, A.; LORENTE S. The constructal law and the evolution of design in nature. **Physics of Life Review**, v.. 8, p. 209–240, 2011.

[8] BISERNI, C.; ROCHA, L.A.O.; STANESCU, G.; LORENZINI, E. Constructal H-shaped cavities according to Bejan's theory. **International Journal of Heat and Mass Transfer**, v. 50, p. 2132–2138, 2007.

[9] LORENZINI, G.; BISERNI, C.; ROCHA, L.A.O. Geometric optimization of isothermal cavities according to Bejan's theory. **International Journal of Heat and Mass Transfer**, v. 54, p 3868–3873, 2011.

[10] ROCHA, L.A.O.; LORENZINI, E.; BISERNI, C. Geometric optimization of shapes on the basis of Bejan's constructal theory. **International Comm. Heat and Mass Transfer**, v. 32, p.1281–1288, 2005.

[11] ROCHA, L.A.O.; LORENZINI, G.; BISERNI, C.; CHO, Y. Constructal design of a cavity cooled by convection. **International Journal of Design & Nature and Ecodynamics**, v. 3, p. 212–220, 2010.

[12] **MATLAB User's Guide**, Version 6.0.088, Release 12, The Mathworks Inc., Natick, MA, 2000.

6. ACKNOWLEDGEMENTS

During the course of this work Mrs. Roberta's work was supported by CAPES and Prof. Rocha's work was supported by CNPq, Brasília, DF, Brasil. E. D. dos Santos thanks FAPERGS by financial support (Process: 12/1418-4).