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NUMERICAL STUDY OF RESIN DISTRIBUTION IN TWO DIFFERENT ARRANGEMENTS OF VASCULAR CHANNELS BY MEANS OF CONSTRUCTAL DESIGN

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Abstract. *In the present work two different arrangements of vascular channels are studied numerically and their geometry is optimized by means of Constructal Design. The main purpose is to seek for the best geometry which minimizes the resin flow resistance inside the channels. The arrangement of vascular channels consists in two horizontal channels of diameter D_2 connected with two vertical channels of diameter D_1. The channels of resin flow are distributed in a solid domain with two different ratios of height and length (H/L = 0.67 and 1.5) in order to illustrate the process of regeneration of composite materials. For all of evaluated configurations the ratio between the areas occupied by the channels and by the solid domain are kept fixed (ϕ = 0.1). It is considered a two-dimensional, laminar and steady state flow (Re_{D2} = 1.0). The conservation equations of mass and momentum are solved numerically by means of the finite volume method (FVM). The results showed that the optimal geometric configuration has a flow resistance several times lower than that found with the worst geometry. For example, for H/L = 0.67, the ratio $(D_1/D_2)_o$ = 0.76 conduct to a fluid dynamic performance nearly 32 times superior than that found for D_1/D_2 = 0.1. It is also noticed that the best shapes are achieved when the pressure and velocity fields has the most homogeneous distribution, i.e., according to the constructal principle of "optimal distribution of imperfections".*

Keywords: *Vascular Channels, Geometric Optimization, Constructal Design, Laminar flow, Resin.*

1. INTRODUCTION

The study of multifunctional materials is a research field that has deserved much attention, especially into the self-healing framework. The composites are generated by the macroscopic combination of two or more materials, which have a recognizable interface among them. The self-healing materials are those that have the ability to reestablish their functional structure in an autonomous form when submitted to some damage, avoiding future collapses (White *et al*., 2001). According to the same authors, these materials mimic the effect of biologic systems which self-regenerates their structure "immediately" after suffering some damage, i.e., bio-inspired systems.

The self-healing mechanism used by vegetal and animal systems is based on the secretion of several fluids in the damaged placement, filling it with the fluid and promoting its regeneration (White *et al*., 2001; Terriault *et al*., 2003). One example can be seen in human structure, where the healing of a broken bone is performed by the transport of biological fluids with coagulants and nutrients to the affected region, allowing the generation of cartilaginous fibers which are calcified. This process generates a new bone with an undistinguishable tissue in comparison with the original one. Into the engineering realm, this kind of material is important for several applications, such as: aeronautical industry, medicine, orthopedic prostheses and railway, naval and automobile buildings (White *et al*., 2001; Terriault *et al*., 2003).

One form to self-recovery of damaged material consists on the insertion of microcapsules in a fluid which flows inside channels intruded into the structure to be repaired. One of limitations of this method is the lack of control over the amount of healing agent consumed in the local damaged. In this sense, several empties can be generated inside the material structure due to the propagation of cracks along the solid domain which are not repaired by the healing fluid (Terriault *et al*., 2003). One possibility to minimize these limitations is the transport of microcapsules in vascular channels. In this sense, the search for the best geometry of these channels (diameters and arrangement, for example)

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inside the structure domain is also an important subject and has been studied with the furtherance of Constructal Theory (White *et al*., 2001; Therriault *et al*., 2003; Wang *et al*., 2006; Lee *et al*., 2008).

Constructal Theory is based on Constructal law, which is a physical principle that can be intended as a generalization of the tendency of all things to flow along paths of minimal resistance. Nature and engineering are united in the search for optimized flow architecture. This law has been employed for several applications in all the domains of design generation and evolution, from biology and physics to social organization, technology evolution, sustainability and engineering (Bejan, 2000; Bejan and Lorente, 2008; Bejan and Zane, 2012; Rocha *et al*., 2013).

In the present numerical study Constructal law is used to evaluate the geometrical configuration of two different arrangements of vascular channels, which mimics the transport of self-healing fluid over one solid domain that requires to be regenerated. The main purpose is to seek for the best geometry which minimizes the resin flow resistance inside the channels, i.e., the geometry that will led to the best distribution of self-healing fluid over the structure of solid domain. The arrangement of vascular channels consists in two horizontal channels of diameter D_2 connected with two vertical channels of diameter D_1. The channels of resin flow are distributed in a solid domain with two different ratios of height and length (H/L = 0.67 and 1.5). These configurations are similar to those studied analytically by Kim *et al*. (2007). For all of evaluated configurations the ratio between the areas occupied by the channels and by the solid domain are kept fixed (ϕ = 0.1). It is considered a two-dimensional, laminar and steady state flow (Re_{D2} = 1.0). The conservation equations of mass and momentum are solved numerically by means of the finite volume method (FVM) (Patankar, 1980; Versteeg and Malalasekera, 1995; Maliska, 2004). More precisely, it is employed the commercial software FLUENT™ (Fluent, 2007). The numerical method is evaluated by means of comparison with analytical results of archival literature.

2. MATHEMATICAL AND NUMERICAL MODELING

In the present work, it is solved numerically several two-dimensional (2D) geometries for ducts arrangements similar to that presented in Fig. 1. The vascular channels where the resin flows are represented in Fig. 1 by the dashed area. For all cases, the resin flow is driven by an imposed velocity at the inlet of the channel arrangement in such way that the Reynolds number be equal to unity ($Re_{D2} = \rho V D_2/\mu$ = 1.0). The channel surfaces has the non-slip and impermeability condition ($u = v$ = 0 m/s) and the channel outflow has an atmospheric prescribed pressure.

Concerning the geometrical arrangement, two orthogonal configurations are studied: H/L = 0.67 (whose domain is formed by M = 3 and N = 2 elementary squares of side d in x and y directions, respectively) and H/L = 1.5 (which is formed by M = 2 and N = 3 elementary squares of side d in x and y directions, as illustrated in Fig. 1). In this study, the volume occupied by the channels is kept fixed and several ratios between the diameters of horizontal and vertical channels (D_1/D_2) are investigated. It is worthy to mention that, only the flows inside the channels are investigated here.

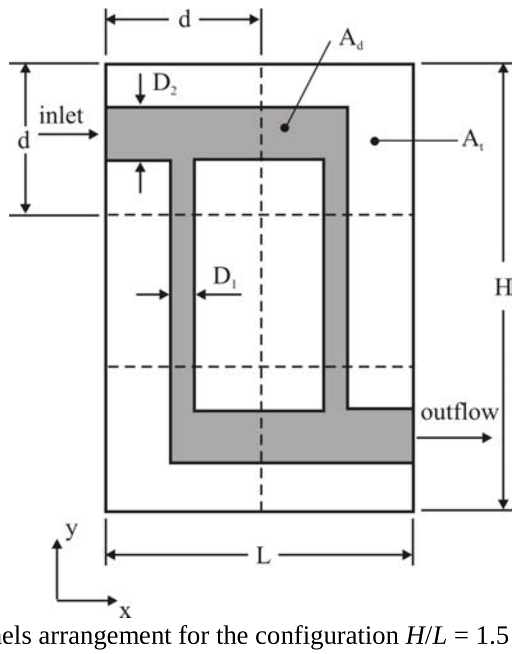

Figure 1. Channels arrangement for the configuration H/L = 1.5 ($M = 2 \times N = 3$).

The main purpose here is to evaluate which ratio of D_1/D_2 minimizes the flow resistance for two configurations of solid domain evaluated (H/L = 0.67 and 1.5). The flow resistance is given by (Kim *et al*., 2007):

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$$R = \frac{\Delta P}{\dot{m}} \tag{1}$$

where ΔP is the pressure difference between the inlet flow and outlet flow (Pa) and $\dot{m}$ represents the mass flow rate of resin inside the channel (kg/s).

As above mentioned, the ratio between the area occupied by the channels (dashed area of Fig. 1, A_d) and the total area of elementar squares (solid domain of area $H \times L$ in Fig. 1, A_t) is given by:

$$\phi = \frac{A_d}{A_t} \tag{2}$$

In this work the area fraction is kept constant (ϕ = 0.1). Concerning the fluid flow, it is considered a resin flow with the following thermo-physical properties: ρ = 1150 kg/m³ and μ = 1.4 kg/(m·s). The inlet velocity is imposed in order to reach to a Reynolds number equal to unity ($Re_{D2} = \rho V D_2/\mu = 1.0$). Other important observation is concerned with the total area, which is considered $H \times L$ = 1 m². In this sense, the elementary dimension $d = (1/6)^{1/2}$ m.

For all simulated cases the flow is considered laminar, incompressible and for a two-dimensional domain. Then, it is required the numerical solution of the conservation equations of mass and momentum in x and y directions, which are given respectively by (Schilichting, 1968):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + X \tag{4}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + Y \tag{5}$$

where υ represents the kinematic viscosity (m²/s), u is the velocity in the x direction (m/s), v is the velocity in the y direction (m/s) and X and Y represents the buoyancy forces in x and y directions (N/m³), respectively. The latter terms are neglected in the present simulations.

The computational domain is discretized in several finite rectangular volumes using the commercial software GAMBIT™ and the conservation equations, Eq. (3) – (5), are solved with the software FLUENT™, which is based on the Finite Volume Method (FVM) (Patankar, 1980; Versteeg and Malalasekera, 1996; Maliska, 2004). The solver is pressure based (coupled of 1st order for pressure and upwind of 2nd order for momentum). Concerning the convergence, the solution is considered converged when the maximal residual of 10^{-6} was achieved for the mass and momentum equations. Moreover, double precision was used for all numerical simulations.

The grid independence is evaluated for the channel geometry with H/L = 0.67 and D_1/D_2 = 0.1 and the appropriate mesh size dimension was determined by successive refinements until the criterion ε, as shown in Tab. 1. The criterion of grid independence is given by:

$$\varepsilon = \left|\frac{\Delta P_i - \Delta P_{i+1}}{\Delta P_i}\right| \le 1.2 \times 10^{-2} \tag{6}$$

where ΔP_{i+1} represents the pressure difference between the inlet and outlet of the vascular channel using the current grid and ΔP_i represents the pressure difference for the previous (coarser) grid.

As depicted in Tab. 1, the independent grid has 432502 rectangular volumes. This mesh will be employed for all vascular channels studied here.

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Table 1. Grid independence as a function of the pressure difference (ΔP) for H/L = 0.67, D_1/D_2 = 0.1 and ϕ = 0.1.

Number of elements	ΔP (Pa)	$\varepsilon \leq 1.20 \times 10^{-2}$
12796	15102.87	5.47×10^{-3}
24780	17509.88	1.37×10^{-3}
51601	19263.35	9.10×10^{-2}
99120	19789.23	2.70×10^{-2}
203479	20266.06	2.40×10^{-2}
432502	20508.15	1.20×10^{-2}

3. RESULTS AND DISCUSSION

Firstly, it is evaluated the effect of the ratio D_1/D_2 in the range ($0.1 \leq D_1/D_2 \leq 1.0$) over the resin flow resistance (R). Figure 2 shows this effect for the ratio of H_1/L_1 = 0.67, ϕ = 0.1 and Re_{D2} = 1.0. In this figure, only the range ($0.6 \leq D_1/D_2 \leq 1.0$) is shown to evidence the optimal region. In Figure 2 it is possible to notice one intermediate point of optimal for $(D_1/D_2)_o$ = 0.76 that minimizes the resin flow resistance. The once minimized resistance obtained for $(D_1/D_2)_o$ is R_m = 453.73. It is worthy to mention that, the subscripts "o" and "m" means once optimized geometry and once minimized resistance, respectively. The minimal magnitude of R (R_m) is nearly 32 times lower than that obtained for the lower extreme of ratio D_1/D_2 (D_1/D_2 = 0.1) and approximately 7 % lower than that achieved for the upper extreme of D_1/D_2 (D_1/D_2 = 1.0). This behavior is in accordance with the analytical predictions of Bejan and Lorente (2004) for this configuration.

In order to investigate the causes for this behavior, the pressure field distribution for three configurations investigated here: D_1/D_2 = 0.1 (lower extreme), $(D_1/D_2)_o$ = 0.76 and D_1/D_2 = 1.0 (upper extreme) are depicted in Fig. 3(a), (b) and (c), respectively. For lower ratios of D_1/D_2, Fig. 3(a), the flow resistance is higher than the optimal geometry, $(D_1/D_2)_o$, due to the large stricture in the vertical channels. In this sense, it is generated a non-uniform distribution of pressure field, with a step variation from the horizontal channels of diameter D_2 to the vertical ones. For the upper extreme ratio of D_1/D_2 = 1.0, Fig. 3(c) the pressure distribution is more homogeneous in comparison with the case seen in Fig. 3(a) being only slightly poor distributed in comparison with the optimal one, Fig. 3(b). This results shown that the optimal shape will be achieved for the most homogeneous distribution of pressure, i.e., in agreement with the constructal principle of "optimal distribution of imperfections". Figure 4 shows the velocity magnitude field for the same cases: D_1/D_2 = 0.1, Fig. 4(a), $(D_1/D_2)_o$ = 0.76, Fig. 4(b) and D_1/D_2 = 1.0. The velocity fields corroborate the previous findings obtained for the pressure fields. It is also clearly exhibited the poor distribution of velocity for the lower case, while for the other cases it is noticed a better distribution of velocity. The velocity distribution obtained for D_1/D_2 = 1.0, Fig. 4(c), is slight suppressed in the vertical channels in comparison with the optimal configuration, $(D_1/D_2)_o$, leading to a increase of resin flow resistance.

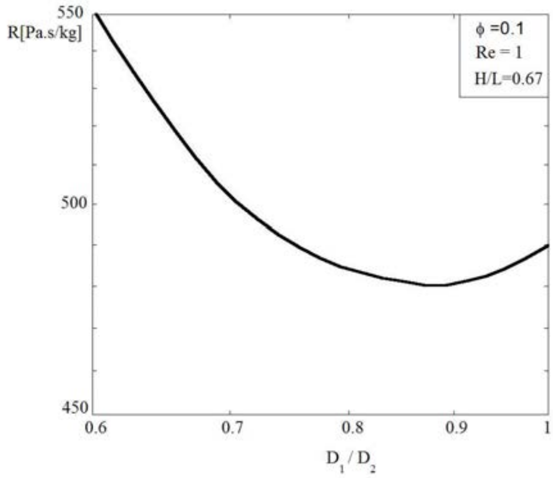

Figure 2. The effect of the ratio D_1/D_2 over the resin flow resistance (R) for the fixed ratio of H/L = 0.67.

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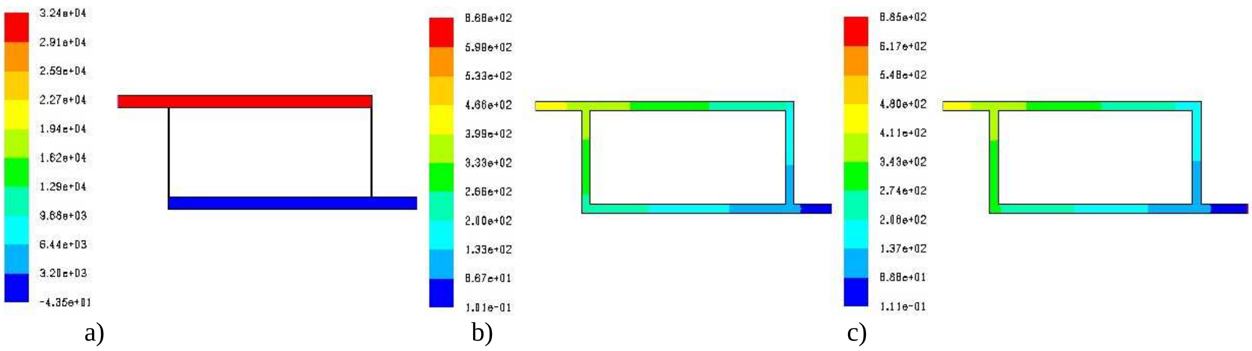


Figure 3. The pressure field distribution in the vascular channels with Re_{D2} = 1.0, ϕ = 0.1, H/L = 0.67 and: a) D_1/D_2 = 0.1 (lower extreme), b) $(D_1/D_2)_o$ = 0.76 and c) D_1/D_2 = 1.0 (upper extreme).

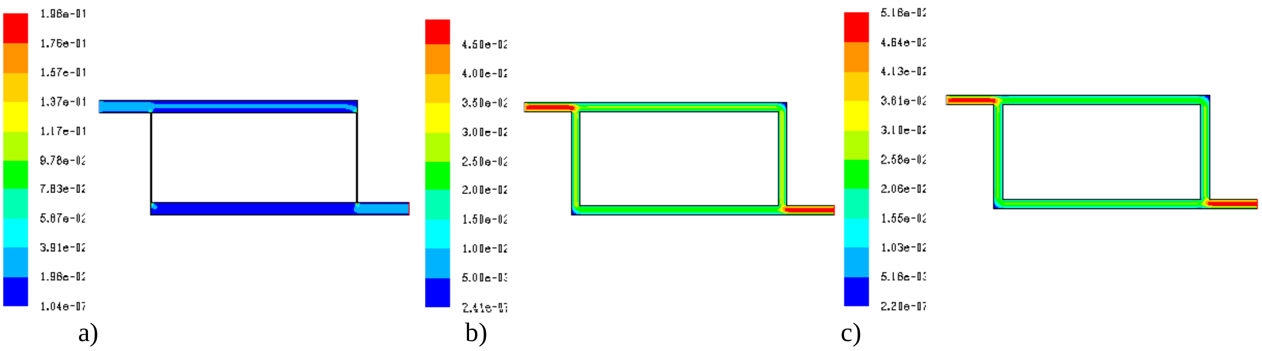


Figure 4. The pressure field distribution in the vascular channels with Re_{D2} = 1.0, ϕ = 0.1, H/L = 0.67 and: a) D_1/D_2 = 0.1 (lower extreme), b) $(D_1/D_2)_o$ = 0.76 and c) D_1/D_2 = 1.0 (upper extreme).

The same procedure was repeated for the optimization of vascular channel with ϕ = 0.1, Re_{D2} = 1.0 and H/L = 1.5. Figure 5 shows the effect of D_1/D_2 over the resin flow resistance (R). It can be observed a similar behavior to that reached for the previous case (H/L = 0.67). However, the point of optimal is displaced from $(D_1/D_2)_o$ = 0.76 for H/L = 0.67 to $(D_1/D_2)_o$ = 0.881 for H/L = 1.5. The optimal shape performs 48 times better than the worst arrangement achieved for D_1/D_2 = 0.1 and nearly 3.0 % better than the upper extreme value of D_1/D_2 investigated (D_1/D_2 = 1.0). Other important observation is that the results reveal the no existence of an universal shape for the vascular channels arrangement that minimizes the resin flow resistance. The shape for the vascular channels adapted to the domain available for occupation, similarly to what is naturally performed by the rivers in the background, i.e., the same mechanism used in nature for the best distribution of fluid flow.

Figures 6 and 7 shows the pressure and temperature field, respectively, for the three ratios of D_1/D_2: D_1/D_2 = 0.1 (the lower extreme), $(D_1/D_2)_o$ = 0.881 and D_1/D_2 = 1.0 (the upper extreme). Similarly to noticed for the case with H/L = 0.67, for this case (H/L = 1.5) the best performance is achieved when the pressure and velocity fields are distributed in the most homogeneous form, i.e., the geometry which facilitates the access of the internal currents.

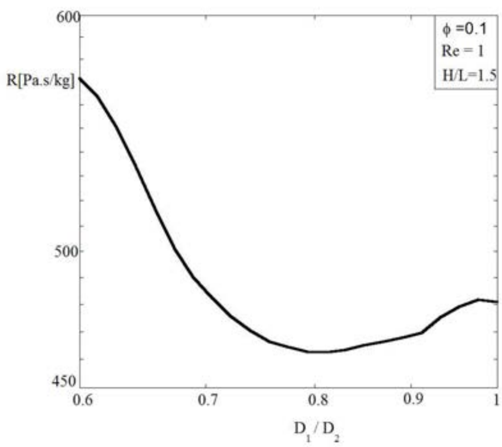
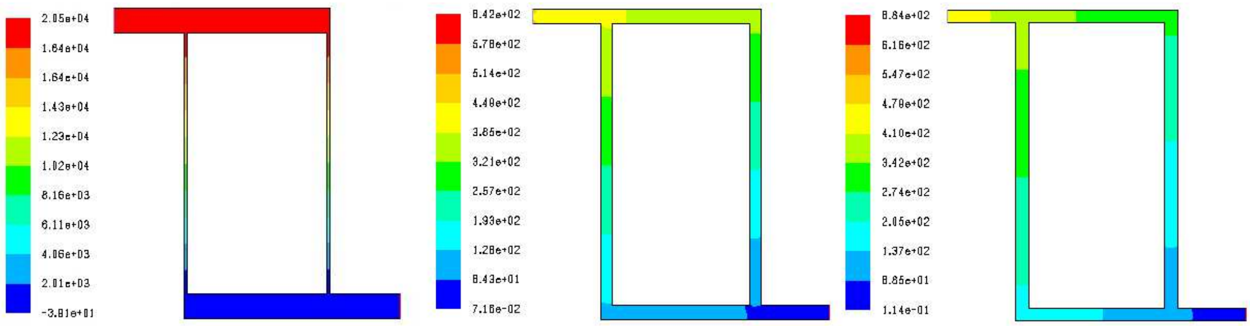


Figure 5. The effect of the ratio D_1/D_2 over the resin flow resistance (R) for the fixed ratio of H/L = 1.5.

4. CONCLUSIONS

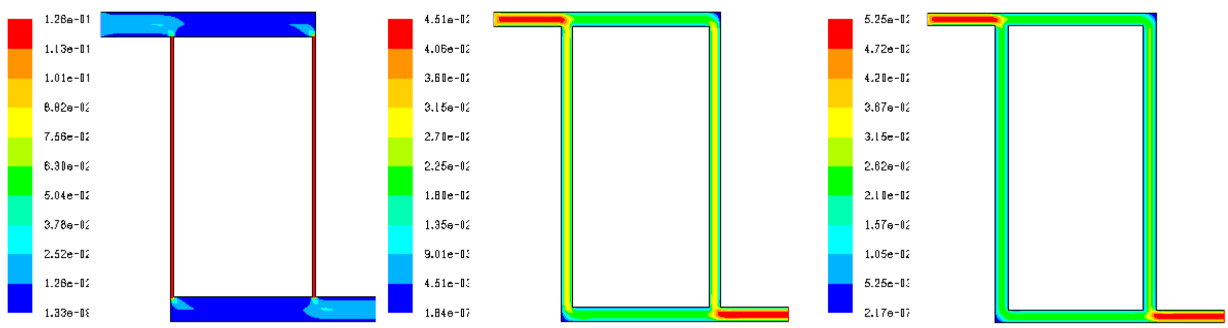
Two different arrangements of vascular channels were numerically studied here and their geometry was optimized by means of Constructal Design, mimicking the resin flow of regeneration fluid in composite materials. The arrangement of vascular channels consisted in two horizontal channels of diameter D_2 connected with two vertical channels of diameter D_1. The channels of resin flow were distributed in a solid domain with two different ratios of H/L: H/L = 0.67 and 1.50, and the ratio of D_1/D_2 was optimized for both configurations. The optimal ratio was that one which minimizes the resin flow resistance. For all of evaluated configurations the ratio between the areas occupied by the channels and by the solid domain was kept fixed (ϕ = 0.1). It was considered a two-dimensional, laminar and steady state flow (Re_{D2} = 1.0). The conservation equations of mass and momentum were solved numerically by means of the finite volume method (FVM).

The results showed that the optimal geometric configuration has a flow resistance several times lower than that found with the worst geometry. For H/L = 0.67, the ratio $(D_1/D_2)_o$ = 0.76 conduct to a fluid dynamic performance nearly 32 times superior than that found for D_1/D_2 = 0.1 (the worst case). For H/L = 1.50, the optimal ratio $(D_1/D_2)_o$ = 0.881 is almost 48 times higher than those found for the worst performance, D_1/D_2 = 0.1. The best shapes were achieved when the pressure and velocity fields has the most homogeneous distribution, i.e., according to the constructal principle of "optimal distribution of imperfections". The results also revealed the no existence of an universal shape for the vascular channels arrangement that minimizes the flow resistance. In other words, the shape adapted to the domain available for occupation – the same mechanism used in nature for the best distribution of fluid flow, e.g., the distribution of river in the background.



a) b) c)

Figure 6. The pressure field distribution in the vascular channels with Re_{D2} = 1.0, ϕ = 0.1, H/L = 1.5 and: a) D_1/D_2 = 0.1 (lower extreme), b) $(D_1/D_2)_o$ = 0.76 and c) D_1/D_2 = 1.0 (upper extreme).



a) b) c)

Figure 7. The velocity field distribution in the vascular channels with Re_{D2} = 1.0, ϕ = 0.1, H/L = 1.5 and: a) D_1/D_2 = 0.1 (lower extreme), b) $(D_1/D_2)_o$ = 0.76 and c) D_1/D_2 = 1.0 (upper extreme).

5. ACKNOWLEDGEMENTS

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