CONSTRUCTAL DESIGN OF PERFORATED STEEL PLATES SUBJECT TO LINEAR ELASTIC AND NONLINEAR ELASTO-PLASTIC BUCKLING

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Abstract. *Steel plates are used in a great variety of engineering applications, such as deck and bottom of ship structures, and platforms of offshore structures. Cutouts are often provided in plate elements for inspection, maintenance, and service purposes. So, the design of shape and size of these holes is significant. Usually these plates are subjected to axial compressive forces which make them prone to instability or buckling. If the plate is slender, the buckling is elastic. However, if the plate is sturdy, it buckles in the plastic range causing the so-called inelastic (or elasto-plastic) buckling.Therefore, the goal of this work is to obtain the optimal geometry which maximizes the buckling load for steel plates with a centered elliptical perforation when subjected to linear and nonlinear buckling phenomenon by means of Constructal Design. To do so, numerical models were developed in ANSYS software to evaluate the elastic and elasto-plastic buckling loads of simply supported and uniaxially loaded rectangular plates with elliptical cutouts. The results indicated that the optimal shapes were obtained in accordance with the Constructal Principle of "Optimal Distribution of*

Imperfections", showing that the Constructal Design method can be satisfactorily employed in mechanic of materials problems.

Keywords: *Constructal Design, Perforated Steel Plates, Linear Elastic Buckling, Nonlinear Elasto-Plastic Buckling, Numerical Simulation*

1 INTRODUCTION

In the analysis of the mechanical behavior of slender members, equilibrium and compatibility conditions are used in order to find the internal forces and deformations. In the simplest cases, a structure's safety is evaluated by confirming that the maximum values computed for the stresses are lower than the allowable stress defined for the material the structure is made of. This is a necessary condition for structural safety, but it may not be sufficient, either because the deformations are limited for some reason, or because there is the risk that the equilibrium configuration of the structure is not stable, i.e., that buckling may occur. In fact, while tensile forces may only do work if the material deforms or ruptures, for the case of compression there is a third possibility ‑ buckling ‑ which consists of a lateral deflection of the material, in relation to direction of actuation of the compressive forces. In accordance with these considerations, the stability of a structure may be analyzed by computing its critical load, i.e., the load corresponding to the situation in which a perturbation of the deformation state does not disturbs the equilibrium between the external and internal forces (Silva, 2006).

In this context, it is well known that steel plate elements constitute very important structural components in many structures, such as ship grillages and hulls, dock gates, plate and box girders of bridges, platforms of offshore structures, and structures used in aerospace industries. In many cases, these plates are subjected to axial compressive forces which make them prone to instability or buckling. If the plate is slender, the buckling is elastic. However, if the plate is sturdy, it buckles in the plastic range causing the so-called inelastic (or elasto-plastic) buckling (El-Sawy et al., 2004).

Besides, in several practical situations cutouts are provided in plate structures for the purposes of access, services and even aesthetics. The presence of these holes results in a redistribution of the membrane stresses accompanied by a change in mechanical behaviors of the plates. Concretely, a significant reduction in elasto-plastic ultimate strength, when compared to solid plate (i.e., imperforated plate), has always been found in perforated plates notwithstanding the occasionally occurring increase in elastic buckling critical load as reported in previous articles (Cheng & Zhao, 2010).

Among the elastic buckling studies category, El-Sawy & Nazmy (2001) investigated the effect of aspect ratio on the elastic buckling critical loads of uniaxially loaded rectangular plates with eccentric circular and rectangular (with curved corners) holes. El-Sawy and Martini (2007) used the finite element method to determine the elastic buckling stresses of biaxially loaded perforated rectangular plates with longitudinal axis located circular holes. Alternatively, Moen & Schafer (2009) developed, validated and summarized analytical expressions for estimating the influence of single or multiple holes on the elastic buckling critical stress of plates in bending or compression. In Rocha et al. (2012), Isoldi et al. (2013) and Rocha et al. (2013) the Constructal Design method was employed to determine the best shape and size of centered cutout in a plate, aiming to maximize the critical buckling load.

In the group of studies dedicated to the problem of elasto-plastic buckling, El-Sawy et al. (2004) investigated the elasto-plastic buckling of uniaxially loaded square and rectangular plates with circular cutouts by the use of the finite element method, including some recommendations about hole size and location for the perforated plates of different aspect ratios and slenderness ratios. Afterwards, Paik (2007a, 2007b, 2008) studied the ultimate strength characteristics of perforated plates under edge shear loading, axial compressive loading and the combined biaxial compression and edge shear loads, and proposed closed-form empirical formulae for predicting the ultimate strength of perforated plates based on the regression analysis of the nonlinear finite element analyses results. Maiorana et al. (2008, 2009) focused on the linear and nonlinear finite element analyses of perforated plates subjected to localized symmetrical load.

Therefore, it is obvious that studies to better understand the mechanical behavior of steel perforated plates has a fundamental importance in structural engineering, especially if the focus is to improve the performance of these structural elements. Hence, the main purpose of the present work is to improve the mechanical behavior of steel perforated plates by means the Constructal Design method.

The Constructal Design method is based on the Constructal Theory which states that: “for a flow system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the currents that flow through it”. The Constructal-law field started from the realization that “design” is a universal physics phenomenon (Bejan & Lorente, 2013). Constructal law can be intended as a generation of the tendency of all things to flow along paths of minimal resistance. Moreover, this physical principle unites the animate with inanimate over an extremely broad range of flow systems. As a consequence, it has been employed for several applications in all the domains of design generation and evolution, from biology and physics to social organization, technology evolution, sustainability and engineering (Bejan &Lorente, 2008; Bejan & Zane, 2012). Most of the activity in the field of constructal theory and design has been devoted to the development of architectures for fluid flow and heat transfer. However, it is possible to consider the solid structures as flow systems that are configured and morph so that they facilitate the flow of stresses. To look at stresses as flow is quite unusual but it is effective when the objective is to discover the best configuration of the stressed volume (Lorente et al., 2010; Isoldi et al., 2013).

Thus, numerical models based on the Finite Element Method (FEM) were used to evaluate the linear elastic and the nonlinear elasto-plastic buckling load of several geometries proposed by the Constructal Design of a simply supported and uniaxially loaded rectangular plate with centered elliptical cutout (Fig. 1). To do so, the degree of freedom (DOF) H_0/L_0 (ratio between the characteristic dimensions of the elliptical hole) is varied for two fixed value of the DOF H/L (ratio between height and length of the plate) of 0.5 and 1.0. The constraints are the hole volume fraction (ϕ), which is the ratio between the perforation volume and the total plate volume (without perforation), and the plate slenderness (H/t), defined by the ratio between height and thickness of the plate. Values of $\phi = 0.2$, $H/t = 50$ and $H/t = 100$ are adopted. Accordingly, the objective function is to maximize linear elastic and the nonlinear elasto-plastic buckling load among the studied cases.

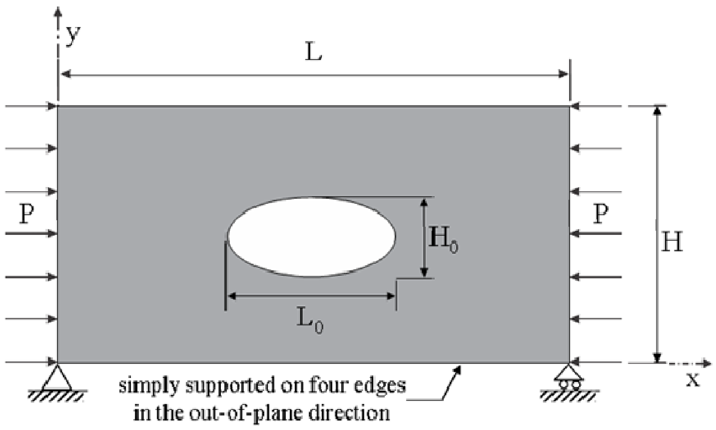

Figure 1. Geometry and loading of a plate with elliptical

2 BUCKLING OF PLATES

Plate elements are often subjected to normal and shearing forces acting in the plane of the plate. If these in-plane forces are sufficiently small, the equilibrium is stable and the resulting deformations are characterized by the absence of lateral displacements (out of plane). As the magnitude of these in-plane forces increases, at a certain load intensity, a marked change in the character of the deformation pattern takes place. That is, simultaneously with the in-plane deformations, lateral displacements are introduced. In this condition, the originally stable equilibrium becomes unstable and the plate is said to have buckled. The load producing this elastic (linear) buckling is called the critical load (P_{cr}). The importance of the critical load is the initiation of a deflection pattern that, if the load is further increased, rapidly leads to very large lateral deflections, so-called the elasto-plastic (nonlinear) buckling, and eventually to complete failure of the plate. This is a dangerous condition that must be avoided (Szilard, 2004). Therefore, plate buckling has a post-critical load-carrying capacity that enables for additional loading after elastic buckling has occurred. A plate is in that sense inner statically indeterminate, which makes the collapse of the plate not coming when elastic buckling occurs, but instead later, at a higher loading level reached in the elasto-plastic buckling. This is taken into consideration in the ultimate limit state design of plates because the elastic buckling does not restrict the load carrying capacity to the critical buckling stress, instead the maximum capacity consists of the two parts: the buckling load added to the additional post-critical load (Åkesson, 2007). In other words, the ultimate loading capacity (P_u) of plates is not restricted to the occurrence of elastic buckling once these structural elements do possess ability for a post-critical reserve strength, which enables for an additional loading capacity after that buckling has occurred. This post-critical reserve strength is shown in the load/displacement diagram in Fig. 2.

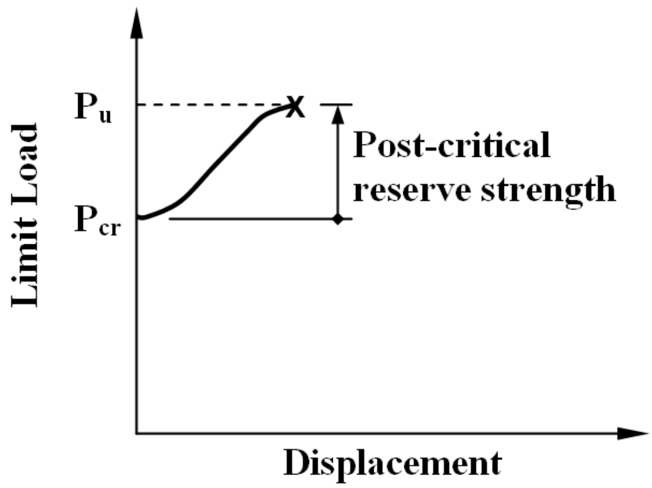

Figure 2. Load/displacement diagram in the post-critical range

This capacity to carry additional load after elastic buckling is due to the formation of a membrane that stabilizes the buckle through a transverse tension band. When the central part of the plate buckles, it loses the major part of its stiffness, and then the load is forced to be “linked’’ around this weakened zone into the stiffer parts on either side. And due to this redistribution a transverse membrane in tension is formed and anchored, as can be seen by the load paths in Fig. 3.

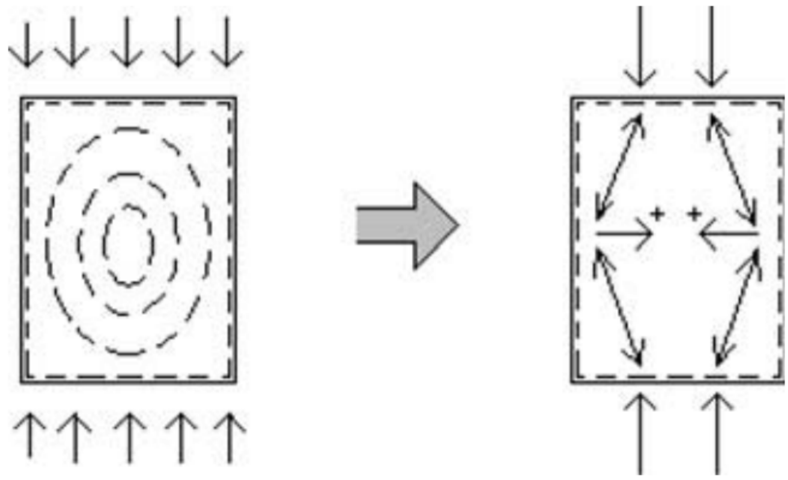

Figure 3. The redistribution of the transfer of load in the ultimate limit state (Åkesson, 2007)

The relative magnitude of the post buckling strength to the buckling load depends on various parameters such as dimensional properties, boundary conditions, types of loading, and the ratio of buckling stress to yield stress (Yoo & Lee, 2011).

The use of this additional strength is of great practical importance in the design of ship and aerospace structures, since by considering the post buckling behavior of plates, considerable weight savings can be achieved. In these structures, the edges of the plates are usually supported by stringers in such a way that they remain straight. This construction practice permits the use of higher than critical loads as allowable edge forces, even under service conditions (Szilard, 2004).

3 COMPUTATIONAL MODELS

Many problems in structural analysis are governed by differential equations. The solutions to these equations would provide an exact, closed-form solution to the particular problem being studied. However, such analytical solutions are only available for problems involving very simple geometry, loading and boundary conditions. Hence, for a more complex problem, the computational modeling can be employed to obtain an approximate solution. This is the situation of the engineering problems addressed in the present work, where the linear elastic buckling load and the nonlinear elasto-plastic buckling load of perforated steel plates need to be numerically evaluated. To do so, computational models developed in the software ANSYS, which is based on the Finite Element Method (FEM), were adopted.

The FEM is a numerical procedure for obtaining approximate solutions to many of the problems encountered in engineering analysis with reasonable accuracy. In the field of structural analysis, the FEM is usually adopted in its displacement formulation. In this way, the structure continuum is divided into a number of small regions – the so-called finite elements. These elements are assumed to be interconnected at a discrete number of nodal points located on their boundaries (Bathe, 1996; Zienkiewicz & Taylor, 1989).

A set of interpolation functions is used to define uniquely the state of displacement within each element in terms of its nodal displacements. The state of strain within the element is uniquely defined by the strain-displacement relationship. The state of stress throughout the element is determined by the material stress-strain law. By applying the Virtual Work Principle, the nodal forces corresponding to a displacement field in the element are determined. These nodal forces are related to the nodal displacements through the element stiffness matrix. Thus, the conditions of overall equilibrium have already been satisfied within the element. Now, all that is necessary is to establish equilibrium conditions at the nodes of the structure. The resulting linear equation system will contain the displacements as unknowns. Once these equations have been solved the structural problem is determined. The internal forces in elements, or the stresses, can easily be found by using the strain-displacement relationship and the material stress-strain law (Real & Isoldi, 2010).

In the present work the 8-Node Structural Shell finite element so-called SHELL93 was used (Fig. 4). This element is particularly well suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal x, y and z directions and rotations about the nodal x, y and z axes. The deformation shapes are quadratic in both in-plane directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities (ANSYS, 2005).

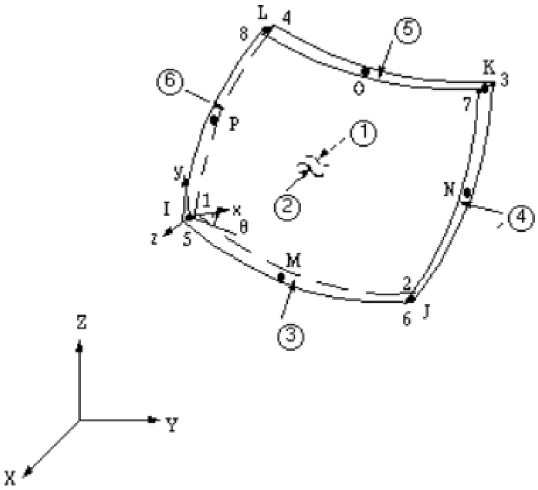

Figure 4. SHELL93 finite element

3.1 Linear Elastic Buckling

Here the approach adopted for the buckling analysis was the eigenvalue buckling. This numerical procedure is used for calculating the theoretical buckling load of a linear elastic structure. Since it assumes the structure exhibits linearly elastic behavior, the predicted buckling loads are overestimated. So, if the component is expected to exhibit structural instability, the search for the load that causes structural bifurcation is referred to as a buckling load analysis. Because the buckling load is not known a priori, the finite element equilibrium equations for this type of analysis involve the solution of homogeneous algebraic equations whose lowest eigenvalue corresponds to the buckling load, and the associated eigenvector represents the primary buckling mode (Madenci and Guven, 2006).

The strain formulation used in the analysis includes both the linear and nonlinear terms. Thus, the total stiffness matrix, [K], is obtained by summing the conventional stiffness matrix for small deformation, [K_E], with another matrix, [K_G], which is the so-called geometrical stiffness matrix (Przemieniecki, 1985). The matrix [K_G] depends not only on the geometry but also on the initial internal forces (stresses) existing at the start of the loading step, $\{P_0\}$. Therefore the total stiffness matrix of the plate with load level $\{P_0\}$ can be written as:

$$[K]=[K_E]+[K_G]. \tag{1}$$

When the load reaches the level of $\{P\}=\lambda\{P_0\}$, where λ is a scalar, the stiffness matrix can be defined as:

$$[K]=[K_E]+\lambda[K_G]. \tag{2}$$

Now, the governing equilibrium equations for the plate behavior can be written as:

$$\big[[K_E]+\lambda[K_G]\big]\{U\}=\lambda\{P_0\} \tag{3}$$

where $\{U\}$ is the total displacement vector, that may therefore be determined from:

$$\{U\}=\big[[K_E]+\lambda[K_G]\big]^{-1}\lambda\{P_0\}. \tag{4}$$

At buckling, the plate exhibits a large increase in its displacements with no increase in the load. From the mathematical definition of the matrix inverse as the adjoint matrix divided by the determinant of the coefficients it is possible to note that the displacements $\{U\}$ tend to infinity when:

$$\det\left[\left[K_E\right]+\lambda\left[K_G\right]\right]=0. \tag{5}$$

Equation (5) represents an eigenvalue problem, which when solved provides the lowest eigenvalue, λ_1, that corresponds to the critical load level $\{P_{cr}\}=\lambda_1\{P_0\}$ at which buckling occurs. In addition, the associated scaled displacement vector $\{U\}$ defines the mode shape at buckling. In the finite element program ANSYS, the eigenvalue problem is solved by using the Lanczos numerical method (ANSYS, 2005).

The verification of this numerical procedure was performed considering a steel (CA-25) plate showed in Fig. 2 with dimensions H = 1000 mm, L = 2000 mm, t = 10 mm, $H_0 = L_0 = 0$ (without perforation). The critical elastic buckling load of a simply supported plate has an analytical solution given by (Åkesson, 2007; Wang et al., 2005):

$$P_{cr}=k\frac{\pi^2Et^3}{12H^2\left(1-\nu^2\right)} \tag{6}$$

where π is the mathematical constant, E and ν are the Young's modulus and the Poisson's ratio of the plate material, respectively, and k is the buckling coefficient, defined as:

$$k=\left(m\frac{H}{L}+\frac{1}{m}\frac{L}{H}\right)^2 \tag{7}$$

being m the number of half waves that occur in the plate's longitudinal direction at buckling, defining the buckling mode of the plate. Being the steel properties E = 210 GPa and ν = 0.3, an analytical critical elastic buckling load of 759.20 kN/m was determined by Eq. (6).

To obtain the numerical solution quadrilateral elements were employed and three different meshes with maximum length size of 10, 20 and 30 mm were adopted. Thus, in addition to making the computational model verification was also performed a mesh independence investigation. The numerical results obtained are presented in Table 1.

Table 1. Numerical critical elastic buckling load used in verification and independence mesh processes

Element size (mm)	P_{cr} (kN/m)
10	753.74
20	753.74
30	753.75

Observing Table 1, there is no significant difference among the numerical results. Hence the mesh with maximum interval size of 20 mm was chosen to be used. Moreover, if this numerical result is compared with the analytical solution a difference of -0.72% is found, verifying the computational model proposed for the analysis of elastic linear buckling behavior of plates.

3.2 Nonlinear Elasto-Plastic Buckling

Determination of the elasto-plastic buckling load of a plate is considerably more difficult than that of its elastic counterpart, since the stress-strain relationship beyond the proportional limit is more complex. Consequently, numerical methods are strongly recommended for stability analysis of plates in the elasto-plastic region (Szilard, 2004).

To do so, the plate material was assumed to be linear elastic–perfectly plastic (i.e., with no strain hardening) which is the most critical case for the steel material. An initial imperfect geometry that follows the buckling mode of an elastic eigenvalue pre-analysis is assumed. The maximum value of the imperfection is chosen to be *H*/2000 (El-Sawy et al., 2004), being *H* the plate width (see Fig. 1).

To find out the plate ultimate load, a reference load given by $P_y = \sigma_y.t$, where σ_y is the material yielding strength, was applied in little increments in the plate edge parallel to the *y* axis. For each load increment the standard Newton-Raphson method was applied to determine the displacements that correspond to the equilibrium configuration of the plate through the equations:

$$\{P\}_{i+1} = \{P\}_i + \{\Delta P\}, \tag{8}$$

$$\{\psi\} = \{P\}_{i+1} - \{F_{NL}\}, \tag{9}$$

$$[K_t]\{\Delta U\} = \{\psi\}, \tag{10}$$

$$\{U\}_{i+1} = \{U\}_i + \{\Delta U\}, \tag{11}$$

where $[K_t]$ is updated tangent stiffness matrix, $\{\Delta U\}$ is the displacements increment vector necessary to reach the equilibrium configuration, $\{F_{NL}\}$is the nonlinear internal nodal forces vector and $\{\psi\}$ is the out-of-balance load vector. The vectors $\{U\}_i$ and $\{U\}_{i+1}$ correspond to the displacements, while the vectors $\{P\}_i$ and $\{P\}_{i+1}$ correspond to the applied external loads at two successive equilibrium configurations of the structure.

If at a certain load stage the convergence could not be achieved; that is, a finite displacement increment cannot be determined so that the out-of-balance load vector $\{\psi\}$ is annulled; it means that the failure load of the structure has been reached. This occurs because no matter as large as the displacements and strains can be, the stresses and internal forces cannot increase as it would be required to balance the external loads. The material has reached the exhaustion of its strength capacity.

In the nonlinear analyses of plates carried out in this paper the reference load was divided in 100 increments and a maximum of 200 iterations was established for each load step. The same discretization used for the elastic linear buckling was adopted to perform the elasto-plastic nonlinear buckling simulations.

4 CONSTRUCTAL DESIGN METHOD

It is possible to state that improving systems configuration for achieving better performance is the major goal in engineering. In the past, the scientific and technical knowledge combined with practice and intuition has guided engineers in the design of man-made systems for specific purposes. Soon after, the advent of the computational tools has permitted to simulate and evaluate flow architectures with many degrees of freedom. However, while system performance was analyzed and evaluated on a scientific basis, system design was kept at the level of art (Bejan & Lorente, 2006).

The Constructal Theory was created by Adrian Bejan, in 1997, when a new geometric solution philosophy was applied to the conductive cooling of electronics (Bejan, 1997; Bejan, 2000). These studies have a significant importance because they played a basic and starting point role for the extension and application of Constructal Theory to problems in engineering and other branches of science (Bejan & Lorente, 2008; Ghodoossi, 2004). Moreover, Constructal Theory has been employed to explain deterministically the generation of shapes in nature (Bejan, 2000).

The lesson taught by the Bejan's Constructal Theory is: geometry matters. The principle is the same in engineering and nature: the optimization of flow systems subjected to constraints generates shape and structure (Bejan, 2000).

So, in order to apply this philosophy the Constructal Design method needs one or more degrees of freedom and constraints to achieve an objective function. Considering the Fig. 1, the DOF H_0/L_0 is freely to vary respecting the vertical limit of $H - H_0$ around 200 mm. Moreover, two defined values for the DOF H/L were considered: 0.5 (H = 1000 mm and L = 2000 mm) and 1.0 (H = 1000 mm and L = 1000 m). A constraint called hole volume fraction, which relates the hole volume (V_0) and total plate volume (V) (without perforation), is also taken into account with a value of 0.2 and given by:

$$\phi = \frac{V_0}{V} = \frac{(\pi H_0 L_0 t)/4}{HLt} = \frac{\pi H_0 L_0}{4HL} \tag{12}$$

where π is the mathematical constant; H_0 and L_0 the characteristic dimensions of hole in y and x directions, respectively; H, L and t are the height, length and thickness of the plate, respectively. The other constraint is the plate slenderness (H/t), being values of 50 (t = 20 mm) and 100 (t = 10 mm) adopted in this study.

5 RESULTS AND DISCUSSION

In all studied cases the numerical simulations were carried out with a mesh generated by quadrilateral elements with maximum length size of 20 mm, being steel (A-25) the material of the perforated plates. Besides, the hole volume fraction ϕ = 0.2 was adopted for all investigated cases. Figure 5 presents the numerical results for the elastic and elasto-plastic limit load of the plate with H/L = 1.0 and H/t = 100 related to the H_0/L_0 variation.

The steel perforated plate analyzed in Fig. 5 has two different behaviors depending on the DOF H_0/L_0 variation. For values of H_0/L_0 lower than 1.50 the plate buckles in the elastic way before to reach the elasto-plastic limit load. However, when H_0/L_0 is larger than 1.50 the elasto-plastic buckling occurs without the plate has suffered the elastic buckling. Besides, observing individually the development of linear and nonlinear plate buckling, it is possible to note that maximum critical buckling load is around 1800 kN/m achieved with H_0/L_0 = 2.50; while a maximum ultimate load of approximately 780 kN/m is reached when H_0/L_0 = 1.50. It is evident that the maximum value of load which can be applied to this plate is 780 kN/m, showing the importance of the elasto-plastic buckling studies associated with the elastic buckling analyses. In this sense, if this maximum value of compressive load is compared with the worst elasto-plastic buckling load of around 506 kN/m obtained with H_0/L_0 = 2.50, an improvement of 54 % in the performance of the plate element was reached.

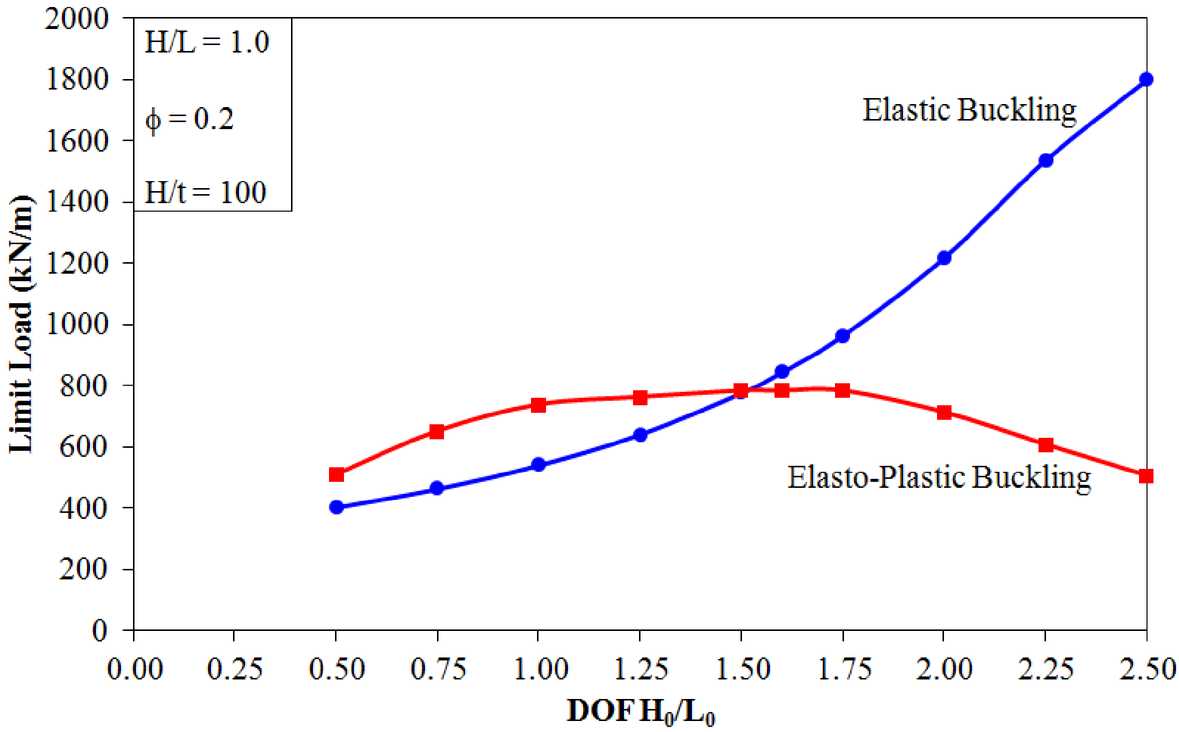

Figure 5. Limit load variation for plate *H*/*L* = 1.0 and *H*/*t* = 100 as function of the DOF H_0/L_0

Another plate considered in this work has the follow characteristics: *H*/*L* = 1.0 and *H*/*t* = 50. The numerical results for its limit load in linear and nonlinear buckling behavior as function of the H_0/L_0 variation is plotted in Fig. 6.

In contrast with the results presented in Fig. 5, the behavior showed in Fig. 6 indicates that for any DOF H_0/L_0 the critical buckling load is larger than the ultimate buckling load, i.e., the plate collapse always occurs before its elastic buckling. In addition, the Constructal Design method was once again able to determine the best shape (H_0/L_0 = 0.50) for the elliptical perforation in the plate achieving a maximum load of 2245 kN/m, which is 122 % greater than the load limit of 1012.50 kN/m obtained for the worst shape (H_0/L_0 = 2.50).

Now, in Fig. 7 are depicted the numerical results for the elastic and elasto-plastic limit load of the steel perforated plate with *H*/*L* = 0.5 and *H*/*t* = 100 related to the H_0/L_0 variation.

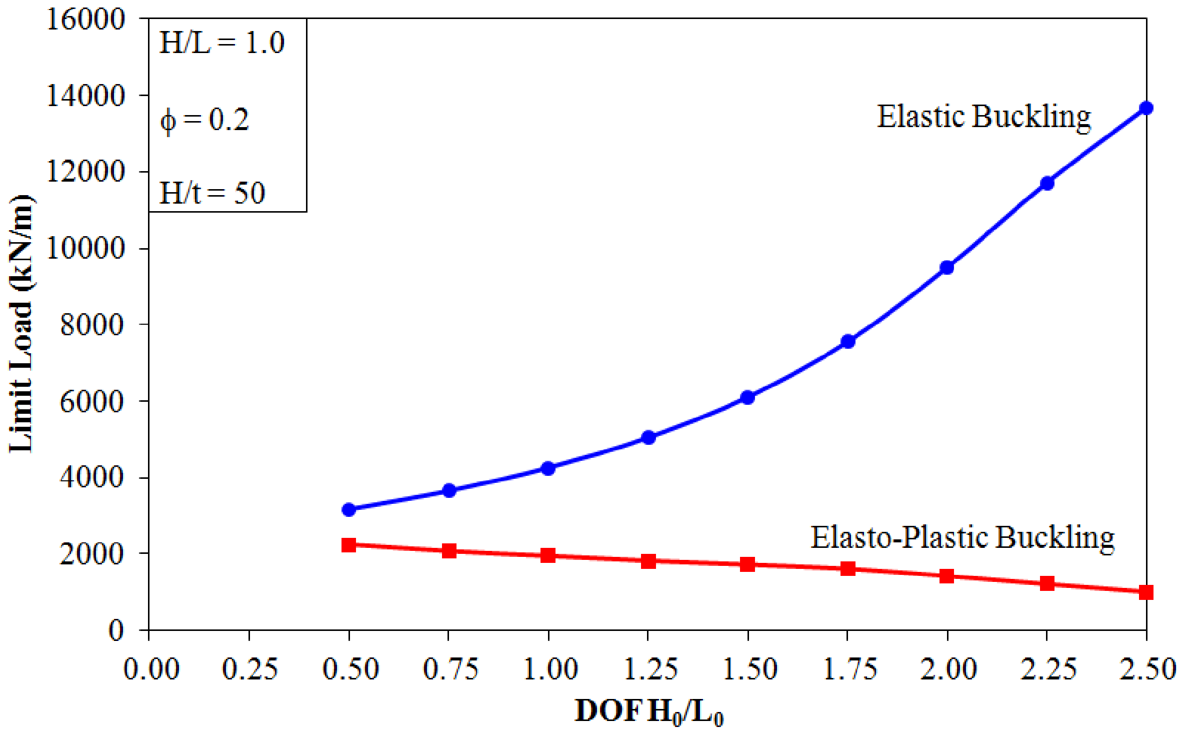

Figure 6. Limit load variation for plate H/L = 1.0 and H/t = 50 as function of the DOF H_0/L_0

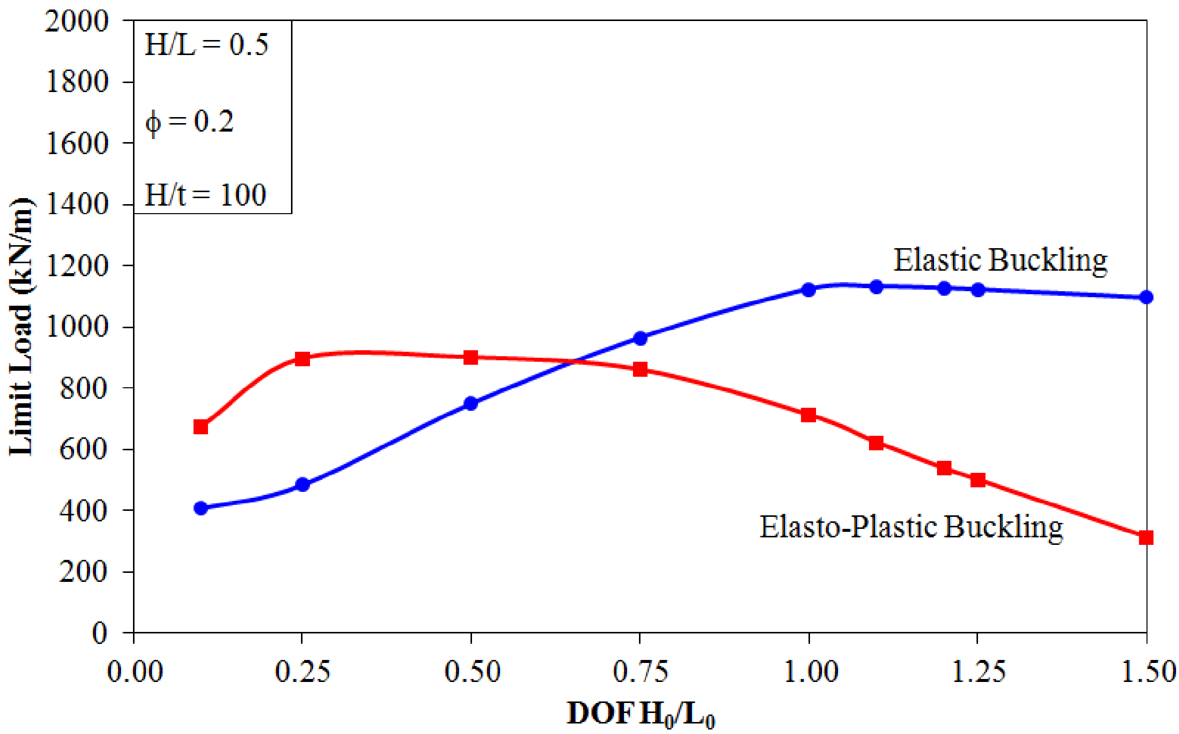

Figure 7. Limit load variation for plate H/L = 0.5 and H/t = 100 as function of the DOF H_0/L_0

Figure 7 shows that when the DOF H_0/L_0 reaches a value around 0.65 the collapse of the plate happens before the occurrence of the elastic buckling. However, for values of H_0/L_0 smaller than 0.65 the plate suffers an elastic buckling before to reach the elasto-plastic limit load (rupture). This behavior trend was also observed in the results of Fig. 5. Moreover, if the elastic and elasto-plastic buckling behavior are separately analyzed one can note that there is a value of H_0/L_0 that conducts to a superior performance, i.e., a maximized limit load. For the linear buckling this best shape is defined by H_0/L_0 = 1.10 and a critical load of almost 1132 kN/m, while for the nonlinear buckling the best shape is obtained for H_0/L_0 = 0.50 with a ultimate load of 900 kN/m. It is important to comment that this last value, in practice, is the maximum compressive load which can be applied to the plate. Before to reach this maximum load, occurs in this plate the elastic buckling in a level of load around 750 kN/m. So, in this case the plate is submitted to the buckling and post buckling behaviors, as showed in Fig. 8.

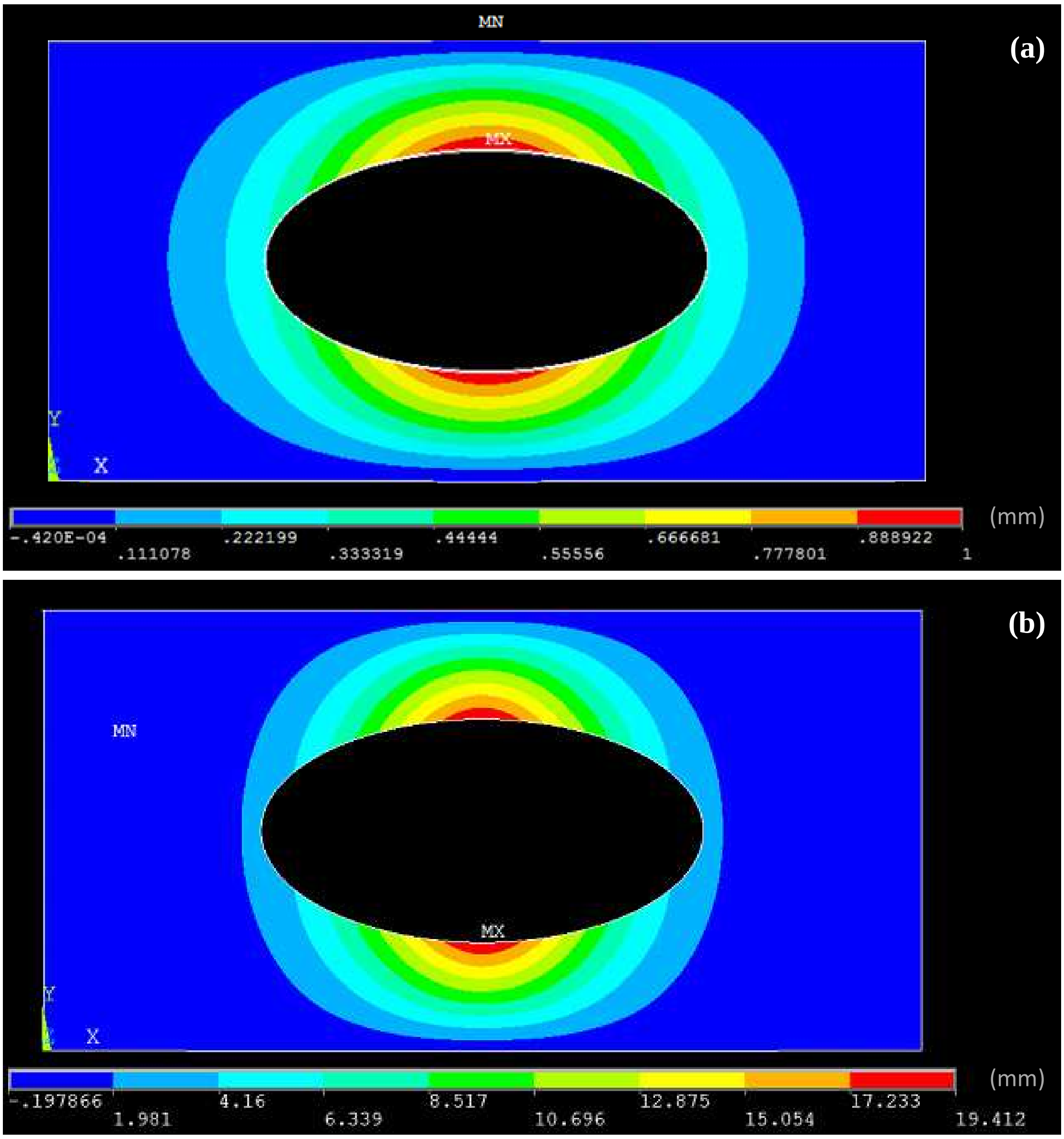

Figure 8. Buckled shape for plate *H*/*L* = 0.5, *H*/*t* = 50 and H_0/L_0 = 0.50 for (a) elastic behavior; (b) elasto-plastic behavior.

Finally, in Fig. 9 is presented the behavior of the elastic and elasto-plastic limit load in accordance with the DOF H_0/L_0 variation, for a plate with H/L = 0.5 and H/t = 50.

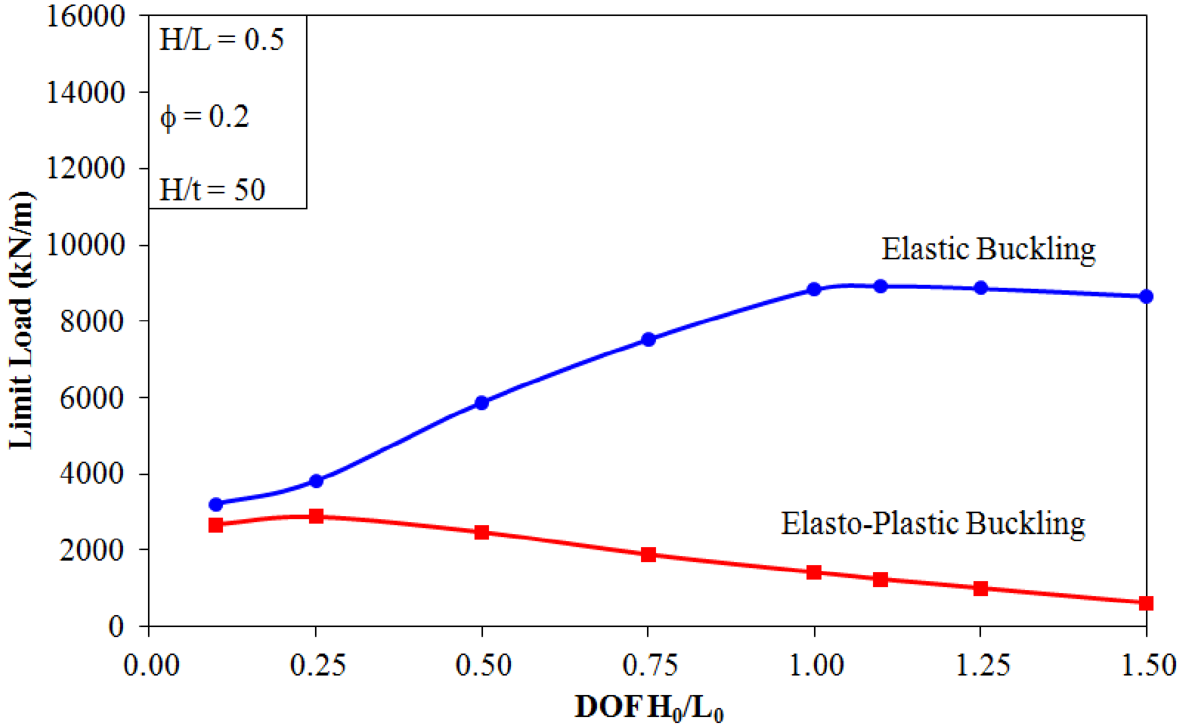

Figure 9. Limit load variation for plate *H/L* = 0.5 and *H/t* = 50 as function of the DOF H_0/L_0

One can note in Fig. 9 that for all values of H_0/L_0 the elasto-plastic buckling occurs before than elastic buckling in the plate, i.e., the plate suffers rupture without suffer the linear buckling phenomenon. However, the same trend observed in Fig. 7 is also observed in Fig. 9: there is a maximum limit load for both buckling types. Moreover, it is possible to observe that in the elasto-plastic buckling results the best performance for the plate is reached for a ultimate load of 2870 kN/m when the H_0/L_0 = 0.25, being this value 359 % superior than the worst performance (H_0/L_0 = 1.50 and ultimate load of 625 kN/m). Figure 10 presents the comparison between these best and worst shapes in the elasto-plastic buckling condition.

It is possible to observe in Fig. 10 that the best shape (Fig. 10a) obeys the Constructal Principle of “Optimal Distribution of Imperfections” once the displacement field has a more uniform distribution than those presented in the worst case (Fig. 10b), validating the employment of the Constructal Design method in mechanic of materials applications.

6 CONCLUSIONS

Plates are structural elements used in several engineering applications as space vehicles, aircraft, buildings and homes, automobiles, bridges decks, submarines, and ships. Hence researches involving the design, behavior analysis, and optimization of these structural components have fundamental importance. In addition, it is well known that in some practical applications the existence of perforations in plates are needed to obtain a reduction of its self-weight or as a way of access through the plate. In this context, the present work studied the elastic linear buckling and the elasto-plastic nonlinear buckling behaviors by means the Constructal Design method associated with the computational modeling.

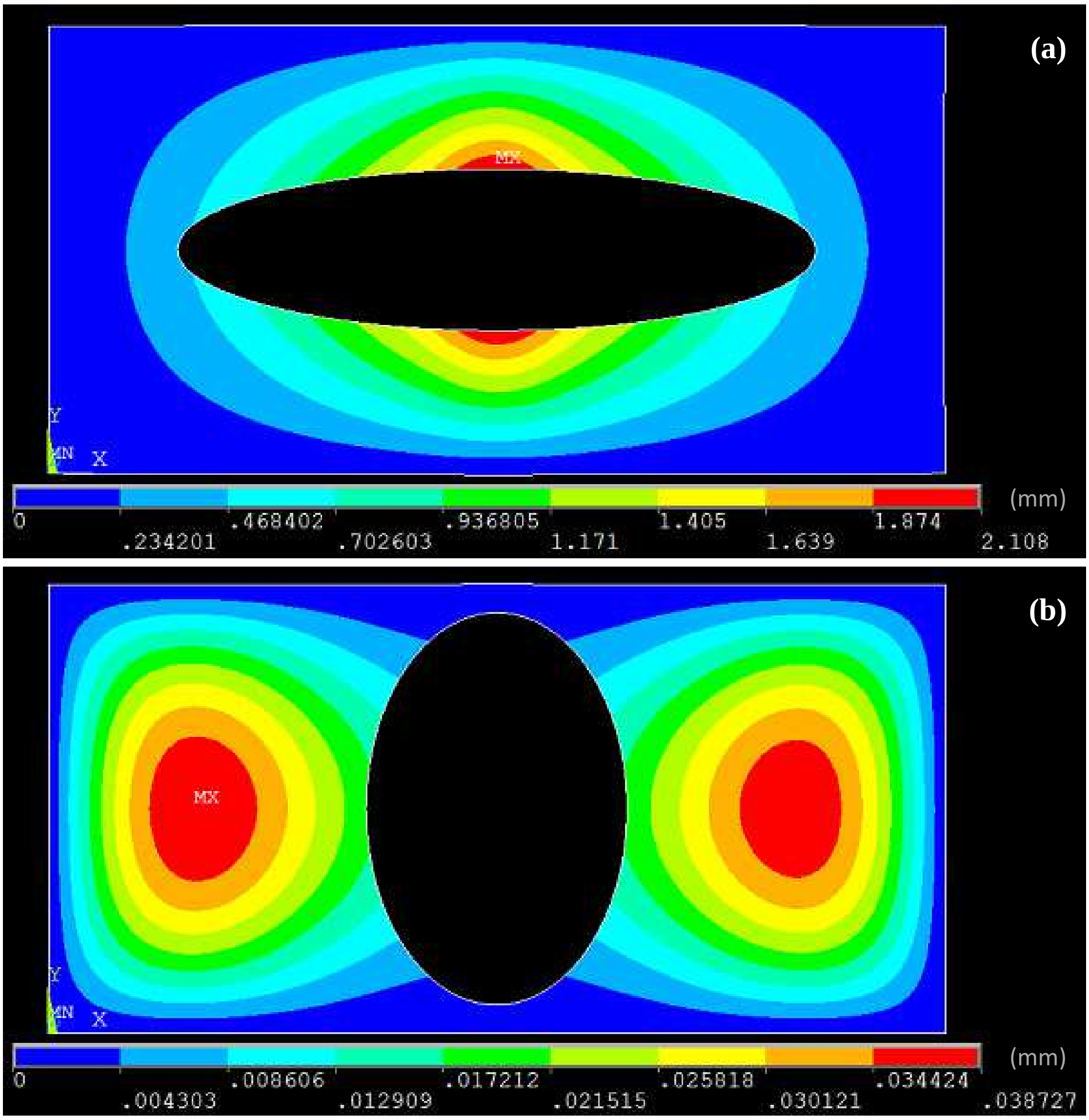

Figure 10. Nonlinear buckled shape for plate *H*/*L* = 0.5, *H*/*t* = 50 and: (a) H_0/L_0 = 0.25; (b) H_0/L_0 = 1.50

To do so, DOF H_0/L_0 (ratio between the characteristic dimensions of the elliptical hole) was varied taking into account two fixed values for the DOF *H*/*L* (ratio between height and length of the plate) of 0.5 and 1.0. The constraints adopted were the hole volume fraction (ϕ), which is the ratio between the perforation volume and the total plate volume (without perforation), and the plate slenderness (*H*/*t*), defined by the ratio between height and thickness of the plate. Values of $\phi = 0.2$, $H/t = 50$ and $H/t = 100$ were used, being the objective function to maximize the limit compressive load which can be uniaxially applied to the steel plate represented in Fig. 1.

The results showed the influence of the DOF H_0/L_0 in the limit load of perforated plates. The critical buckling load that indicates the elastic buckling and the ultimate load defined by the elasto-plastic buckling were maximized due the application of the Constructal Design method in the search of the best shape for the perforated plates.

Moreover, it was possible to observe the importance of the plate slenderness in its buckling behavior. The results indicates that the plate slenderness defines if the plate will suffer the linear elastic buckling before the occurrence of the nonlinear elasto-plastic buckling or if the plate will reach the ultimate load without the happening of elastic buckling. The variation of DOF H_0/L_0 also has influence in this pattern behavior, i.e., the shape of the elliptical perforation can define how the buckling phenomenon will occur.

Finally, the present work demonstrated that the Constructal Design method can be used to improve the performance in mechanic of materials applications once the best shapes are in agreement with Constructal Principle of “Optimal Distribution of Imperfections", justifying the continuity of this work by means the investigation of other values for plate slenderness, hole volume fraction and degree of freedom.

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