Equivalence of Many-Photon Green Functions in DKP and KGF Statistical Quantum Field Theories.

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Abstract

We prove the equivalence of many-photon Green functions in statistical quantum field Duffin-Kemmer-Petiau (DKP) and Klein-Gordon-Fock (KGF) theories using functional path integral formalism for partition functional in statistical quantum (finite temperature) field theory. We also calculate the polarization operators in these theories in one-loop approximation, and demonstrate their coincidence.

1 Introduction

The method of Green functions (GF) in quantum statistics has a long history which begins with Matzubara's work in 1953 [1]. The method of generating or partition functional was first applied for calculation of temperature renormalizable GF in [2], [3] by Fradkin[1]. Later, the method of functional path-integral in statistics was developed in Bernard's work [4].

In the last years, the equivalence of DKP and KGF theories was proved in [8], [9], [10] for the mass shell S-matrix elements of scalar-charged particles interacting with the quantized EM and YM fields, as well as for GF with external photons *off the mass shell*.

An interesting physical question arises in this connection: can one prove the equivalence of many-photon GF in statistical quantum (finite temperature) DKP and KGF theories?

The main goal of this paper is to give positive answer to this question.

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[1] The detailed list of publications may be found in Proceedings of P.N.Lebedev Physical Institute, vol. XXIX, 1965, in Fradkin's dissertation [3], and in Kapusta's [5] and Le Bellac's [6] books. See also references in the paper [7].

In section **2** we give the general proof of the equivalence by path integral method in statistical quantum theory. This result can also be understood from the physical point of view. Photon does not acquire mass and consequently the chemical potential μ too; i.e., photon in temperature medium conserves its nature. As an illustration, in section **3** we calculate polarization operators in one-loop approximation of both theories, and prove that these operators do coincide.

Section **4** contains the conclusions.

2 Coincidence of Many-Photon GF in DKP and KGF at Finite Temperature Theories

To obtain the partition functional $Z(J, \bar{J}, J_\mu)$ in statistical theory one must make transition to Euclidean space and restrict integration on x_4: $0 \leq x_4 \leq \beta$; here $\beta = 1/T$ and $J, \bar{J}, J_\mu$ are external currents. As it follows from general considerations, the partition functional in DKP theory of charged spin-zero particles interacting with the quantized EM field A_μ (in α-gauge) has the following form[2]:

$$Z_{DKP} = Z_0 \int_\beta DA_\mu \, D\psi \, D\bar{\psi} \, \exp\left\{ -\int_0^\beta dx_4 \int_{-\infty}^{\infty} d\mathbf{x} \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 \right.\right.$$
$$\left.\left. + \bar{\psi}(x)(\beta_\mu D_\mu + m)\psi(x) + J_\mu(x) A_\mu(x) + \bar{J}(x)\psi(x) + \bar{\psi}(x) J(x) \right] \right\} \quad (1)$$

where $Z_0 = Z(0,0,0)^3$; $D_\mu = \partial_\mu^* - ieA_\mu$, $\partial_4^* = \partial_4 - \mu$; μ is the chemical potential[4]. In Euclidean space $\bar{\psi}(x) = \psi^*(x)$; all the fields satisfy periodical conditions:

$$\bar{\psi}(0, \mathbf{x}) = \bar{\psi}(\beta, \mathbf{x}), \quad \psi(0, \mathbf{x}) = \psi(\beta, \mathbf{x}), \quad A_\mu(0, \mathbf{x}) = A_\mu(\beta, \mathbf{x}). \quad (2)$$

For example, in Eq. (1):

$$\int_\beta DA_\mu(x) = \int \prod_{0 \leq x_4 \leq \beta} \prod_{-\infty \leq x_i \leq \infty} \prod dA_\mu(x_4, \mathbf{x}). \quad (3)$$

[2] See the paper [4] for the discussion of gauge-dependence.

[3] In the case of charge $e = 0$:

$$Z_0 = \prod_{\mathbf{p},\mathbf{k}} (1 - \exp(-\beta(E - \mu)))^{-1} (1 - \exp(-\beta\omega))^{-1},$$

where $E = \sqrt{\mathbf{p}^2 + m^2}$, $\omega = |\mathbf{k}|$.

[4] In Bose-Einstein case the statistical potential is always negative.

In Euclidean space $\bar{\psi}(x) = \psi^*(x)$; we choose β_μ-matrices in the form:

$$\beta_4 = \begin{vmatrix} \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}, \beta_1 = \begin{vmatrix} \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}, \beta_2 = \begin{vmatrix} \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}, \beta_3 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \end{vmatrix} \tag{4}$$

After the integration over ψ and $\bar{\psi}$ in Eq. (1) we get:

$$Z_{DKP}(\bar{J}, J, J_\mu) = Z_0 \int_\beta DA_\mu(x) \exp\left\{ -\int_\beta d^4x \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 + J_\mu A_\mu \right.\right.$$
$$\left.\left. + \operatorname{Tr}\ln S(x, x, A) \right]^\beta - \int_\beta d^4x d^4y \bar{J}(x) S(x, y, A) J(y) \right\}. \tag{5}$$

Here

$$S(x, y, A) = \left(\beta_\mu D_\mu + m \right)^{-1} \delta^4(x - y) \tag{6}$$

is the GF of a DKP particle in external field $A_\mu(x)$; the term $\operatorname{Tr}\ln S(x, x, A)$ gives rise to all vacuum perturbations diagrams. This term can be transformed into the following component form:

$$\det S(x, y, A) = \int_\beta D\psi D\bar{\psi} \exp\left\{ -\int_\beta d^4x \bar{\psi} \left(\beta_\mu D_\mu + m \right) \psi \right\} =$$
$$= \int_\beta \prod_{\mu=1}^{4} D\phi_\mu D\phi_\mu^* D\phi D\phi \exp\left\{ -\int_\beta d^4x (\phi^* D_\mu \phi_\mu + \phi_\mu^* D_\mu \phi + m(\phi\phi^* + \phi_\mu \phi_\mu^*) \right\}. \tag{7}$$

Now let us integrate over ϕ_μ and ϕ_μ^*. We get:

$$\det S(x, y, A) = \det G(x, y, A) = \exp \operatorname{Tr}\ln G(x, x, A) =$$
$$\frac{1}{m} \int_\beta D\phi D\phi^* \exp\left\{ -\frac{1}{m} \int_\beta d^4x \phi^* \left(-D_\mu^2 + m^2 \right) \phi \right\}, \tag{8}$$

where

$$G(x, y, A) = \left(-D_\mu^2 + m^2 \right)^{-1} \delta^4(x - y) \tag{9}$$

is the GF of the KGF equation in the case of external field $A_\mu(x)$. Thus, we conclude from Eqs. (7–9) that all many-photon GF (not only matrix elements of S-matrix for *real* photons) coincide in DKP and KGF statistical theories[5]. This concludes the proof of equivalence for many-photon GF in KGF and DKP statistical theories.

3 Polarization Operator in One-Loop Approximation

Polarization operator in KGF statistical theory for charged spin-zero scalar particles in one-loop approximation has the form[6]

$$\Pi^k_{\mu\nu}(k) = -\frac{e^2}{(2\pi)^3\beta}\sum_{p_4}\int d\mathbf{p}\left(\frac{(2p+k)_\mu(2p+k)_\nu}{(p^2+m^2)((p+k)^2+m^2)} - \frac{2\delta_{\mu\nu}}{p^2+m^2}\right), \tag{10}$$

where

$$p^2 = p_4^2 + \mathbf{p}^2; \quad p_4 = \frac{2\pi n}{\beta}; \quad -\infty < n < +\infty. \tag{11}$$

The term proportional to $\delta_{\mu\nu}$ in Eq. (10) is important in the proof of transversality of $\Pi_{\mu\nu}$ ($k_\mu\Pi_{\mu\nu}(k) = 0$). However, this term does not contribute to $\Pi_{\mu\nu}$ after the renormalization. In DKP theory, the one-particle GF in momentum space is:

$$\begin{aligned} G(\hat{p}) &= -\frac{1}{m}\left(\frac{i\hat{p}(i\hat{p}+m)}{p^2+m^2}+1\right), \\ \hat{p} &= \beta_\mu p_\mu. \end{aligned} \tag{12}$$

It is easy to check that

$$(i\hat{p}-m)G(\hat{p}) = 1. \tag{13}$$

Using Eqs. (12)–(13) we obtain the polarization operator in DKP theory (in e^2-approximation):

$$\Pi^D_{\mu\nu}(k) = \frac{e^2}{m^2(2\pi)^3\beta}\mathrm{Tr}\sum_{p_4}\int d\mathbf{p}\,\beta_\mu G(\hat{p}+\hat{k})\beta_\nu G(\hat{p}) \tag{14}$$

$$= -\frac{e^2}{(2\pi)^3\beta}\sum_{p_4}\int d\mathbf{p}\left(\frac{(2p+k)_\mu(2p+k)_\nu}{(p^2+m^2)((p+k)^2+m^2)} - \frac{\delta_{\mu\nu}}{p^2+m^2} - \frac{\delta_{\mu\nu}}{(p+k)^2+m^2} + \frac{\delta_{\mu\nu}}{m^2}\right).$$

[5] Strictly speaking, the scalar fields $\phi(x)$ in DKP and $\varphi(x)$ in KGF theory are related by the following equation:

$$\varphi(x) = \frac{1}{\sqrt{m}}\phi(x)$$

[6] The last term in Eq. (10) appears due to the term $e^2\int A^2_\mu(x)\phi^*(x)\phi(x)d^4x$ in the Lagrangian of the KGF theory.

The last term $\sim \delta_{\mu\nu}$ in DKP theory breaks the gauge invariance[7], but disappears after the renormalization. It is easy to show that after the substitution $(p+k) \leftrightarrow p$ in the regularization term $\delta_{\mu\nu}((p+k)^2+m^2)^{-1}$, it will be equal to $\delta_{\mu\nu}(p^2+m^2)^{-1}$. This coincidence of $\Pi^K_{\mu\nu}$ and $\Pi^D_{\mu\nu}$ in one-loop approximation confirms the general proof given in Section 2, see Eqs. (8)–(9)[8].

The $\Pi_{\mu\nu}(k)$ tensor has the form [11], [1]:

$$\Pi_{\mu\nu} = (k_\mu k_\nu - k^2\delta_{\mu\nu})\Pi(k^2). \tag{15}$$

In quantum statistics, $\Pi_{\mu\nu}$ depends on the two vectors: k_μ and u_μ, this latter is the single vector of medium velocity. Thus, in the general case (see [3], p. 75)

$$\begin{aligned}\Pi_{\mu\nu} &= \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)A_1 + \left(u_\mu u_\nu - \frac{k_\mu u_\nu (ku)}{k^2} - \frac{k_\nu u_\mu (ku)}{k^2} + \frac{k_\mu k_\nu (ku)^2}{k^4}\right)A_2 \\ &\equiv \Phi^1_{\mu\nu}A_1 + \Phi^2_{\mu\nu}A_2. \end{aligned} \tag{16}$$

Introducing the notation (for any approximation)

$$a_1 \equiv \Pi_{\mu\mu} = 3A_1 + \lambda A_2 \tag{17}$$

$$a_2 \equiv u_\mu \Pi_{\mu\nu} u_\nu = \lambda(A_1 + \lambda A_2), \qquad \lambda = \left(1 - \frac{(ku)^2}{k^2}\right), \tag{18}$$

we get:

$$A_1 = \frac{1}{2}\left(a_1 - \frac{1}{\lambda}a_2\right), \qquad A_2 = \frac{1}{2\lambda}\left(-a_1 + \frac{3}{\lambda}a_2\right). \tag{19}$$

If the system is at rest[9],

$$\lambda = \left(1 - \frac{k_4^2}{k^2}\right), \tag{20}$$

and

$$a_2 = \Pi_{44} = \left(1 - \frac{k_4^2}{k^2}\right)A_1 + \left(1 - \frac{k_4^2}{k^2}\right)^2 A_2. \tag{21}$$

It is convenient to represent a_1, a_2 in the form:

$$a_i = a_i^R + a_i^\beta. \tag{22}$$

[7] See Bernard's work [4].

[8] One may note that $\Pi^D_{\mu\nu}$ given by Eq. (8.23) in [3] does not coincide with our Eq. (14), breaking the equivalence.

[9] One can show that Eq. (16), strictly speaking, is satisfied only if the system is at *rest* ($\mathbf{u} = 0, u_4 = 1$), see [3], Chapter II, sect. 7.

Here a_i^R are the parts which do not depend on β and which must be renormalizable; a_i^β depend on β. It is important that when the temperature is zero

$$\lim_{\beta\to\infty} a_i^\beta = 0. \tag{23}$$

Now we can write the Eqs. (16) in the following form:

$$\Pi_{\mu\nu} = \frac{1}{2}\Phi^1_{\mu\nu}\left(a_1^R - \frac{1}{\lambda}a_2^R + a_1^\beta - \frac{1}{\lambda}a_2^\beta\right) + \frac{1}{2\lambda}\Phi^2_{\mu\nu}\left(-a_1^R + \frac{3}{\lambda}a_2^R - a_1^\beta + \frac{3}{\lambda}a_2^\beta\right). \tag{24}$$

The term $\sim \Phi^2_{\mu\nu}$ must vanish in the limit $\beta \to \infty$. Therefore in this limit we obtain $\Pi_{\mu\nu}$ of the Euclidean quantum field theory. So far as a_1^R and a_2^R do not depend on β, we conclude that (after the renormalization)

$$a_2^R = \frac{\lambda}{3}a_1^R. \tag{25}$$

Thus

$$\lim_{\beta\to\infty}\Pi_{\mu\nu} = \frac{1}{3}\Phi^1_{\mu\nu}a_1^R \qquad \text{or} \qquad \Pi_{\mu\mu} = a_1^R. \tag{26}$$

We calculate a_1 and a_2 using the general formula for summation over p_4 in Eq. (14). We ignore the terms $\sim \delta_{\mu\nu}$ in Eqs. (10) and (14) since these terms disappear after regularization and renormalization.

$$a_1 = \Pi_{\mu\mu} = -\frac{e^2}{(2\pi)^3\beta}\int d\mathbf{p}\sum_{p_4}\frac{(2p+k)^2}{(p^2+m^2)((p+k)^2+m^2)} \tag{27}$$

$$a_2 = \Pi_{44} = -\frac{e^2}{(2\pi)^3\beta}\int d\mathbf{p}\sum_{p_4}\frac{(2p_4+k_4)^2}{(p^2+m^2)((p+k)^2+m^2)}, \tag{28}$$

where $p_4 = 2\pi n/\beta$.

The general formula for summation over β is[10]

$$\frac{1}{\beta}\sum_n f(\frac{2\pi n}{\beta}, K) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega f(\omega, K) + \frac{1}{2\pi}\int_{-\infty+i\epsilon}^{\infty+i\epsilon} d\omega \frac{f(\omega, K) + f(-\omega, K)}{e^{-i\beta\omega} - 1}. \tag{29}$$

The Eq. (27) can be rewritten in the following form:

$$a_1 = -\frac{e^2}{(2\pi)^3\beta}\int d\mathbf{p}\sum_{p_4}\frac{4p^2 + 4pk + 2k^2 + 4m^2 - (4m^2+k^2)}{(p^2+m^2)((p+k)^2+m^2)}$$

[10] See [3], page 123, supplement 3 and [2], page 299.

$$= -\frac{e^2}{(2\pi)^3\beta}\int d\mathbf{p}\sum_{p_4}\left\{-\frac{4m^2+k^2}{(p^2+m^2)((p+k)^2+m^2)}+\frac{2}{p^2+m^2}+\frac{2}{(p+k)^2+m^2}\right\}$$

$$\Rightarrow +\frac{e^2}{(2\pi)^3\beta}\int d\mathbf{p}\sum_{p_4}\frac{4m^2+k^2}{(p^2+m^2)((p+k)^2+m^2)}, \tag{30}$$

where we omit the last two terms which vanish after renormalization; $p^2 = p_4^2 + \mathbf{p}^2$.

Introducing the Feynman's parameter x into Eq. (30), we obtain:

$$a_1(k^2) = -\frac{e^2}{(2\pi)^3\beta}(4m^2+k^2)\int d\mathbf{p}\frac{\partial}{\partial m^2}\int_0^1 dx\sum_{p_4}\frac{1}{\left[(p^2+m^2)+\frac{k^2}{4}(1-x^2)\right]}. \tag{31}$$

To get the contribution to $a_1(k^2)$ which does not depend on β we must use only the first term of Eq. (29):

$$a_1^R(k^2) = -\frac{e^2}{(2\pi)^3}(4m^2+k^2)\int d\mathbf{p}\frac{\partial}{\partial m^2}\int_0^1 dx\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\left[\omega^2+\mathbf{p}^2+m^2+\frac{k^2}{4}(1-x^2)\right]}. \tag{32}$$

Closing the integration contour at infinity in the upper half-plane, we find:

$$a_1^R(k^2) = -\frac{e^2}{(2\pi)^3}(4m^2+k^2)\int^l d\mathbf{p}\frac{\partial}{\partial m^2}\int_0^1 dx\frac{1}{2\left[\mathbf{p}^2+m^2+\frac{k^2}{4}(1-x^2)\right]^{1/2}}, \tag{33}$$

where l is the momentum cut-off.

After the integration over x and renormalization

$$a_1^R(k^2) \to a_1^R(k^2) - a_1^R(0) - k^2\frac{\partial}{\partial k^2}a_1^R(0), \tag{34}$$

we obtain:

$$a_1^R(k^2) = -\frac{e^2k^4}{16\pi^2}\int_{4m^2}^{\infty}\frac{dz^2\left(1-\frac{4m^2}{z^2}\right)^{3/2}}{z^2(z^2+k^2)}. \tag{35}$$

In the limit $\beta\to\infty$ we get the Euclidean expression for $\Pi_{\mu\nu}$ (see Eq. (26)):

$$\lim_{\beta\to\infty}\Pi_{\mu\nu} = \left(-\frac{k_\mu k_\nu}{k^2}+\delta_{\mu\nu}\right)\left(\frac{e^2}{48\pi^2}\right)k^4\int_{4m^2}^{\infty}\frac{dz\left(1-\frac{4m^2}{z^2}\right)^{3/2}}{z^2(z^2+k^2)}. \tag{36}$$

This expression for $\Pi_{\mu\nu}$ also follows from Eq. (11) in [1], where the photon GF has been calculated in DKP theory using dispersion approach.

One can easily find a_2^R from the Eqs. (25),(35):

$$a_2^R(k^2) = \frac{\lambda}{3}a_1^R = +\frac{e^2k^2}{48\pi^2}(k_4^2 - k^2)\int_{4m^2}^{\infty}\frac{dz\left(1-\frac{4m^2}{z^2}\right)^{3/2}}{z^2(z^2+k^2)}. \qquad (37)$$

One can write the expression[11] for a_1^β and a_2^β ($\mu \neq 0$):

$$a_1^\beta = \frac{e^2}{16\pi^2}(4m^2+k^2)\int_0^\infty \frac{p\,dp}{E|\mathbf{k}|}\left(e^{\beta(E-\mu)}-1\right)^{-1}\ln\frac{(k^2+2p\mathbf{k})^2+4E^2k_4^2}{(k^2-2p\mathbf{k})^2+4E^2k_4^2} \qquad (38)$$

$$a_2^\beta = \frac{e^2}{16\pi^2}\int\frac{p^2\,dp}{Ep|\mathbf{k}|}\left(e^{\beta(E-\mu)}-1\right)^{-1}\left\{(E^2-k_4^2)\ln\frac{(k^2+2p\mathbf{k})^2+4E^2k_4^2}{(k^2-2p\mathbf{k})^2+4E^2k_4^2}\right.$$
$$\left.+\,2iEk_4\ln\frac{(k^2+2iEk_4)^2-4p^2\mathbf{k}^2}{(k^2-2iEk_4)^2-4p^2\mathbf{k}^2}\right\}, \qquad (39)$$

where

$$E = (p^2+m^2)^{1/2}. \qquad (39a)$$

Some details of calculations can be found in the Appendix.

4 Conclusions

We have proved the equivalence of photon GF in DKP and KGF statistical theories (Section **2**), and carried out calculations of polarization operator in one-loop approximation to illustrate the equivalence (Section **3**). It would be interesting to generalize the proof of equivalence for GF of many non-Abelian gluons in statistical quantum field DKP and KGF theories (see [10]).

The generalization of the proof of equivalence for photons GF in DKP and KGF statistical theories to the case of charged vector fields can also be made, however this proof will have a formal character due to non-renormalizability of the theory.

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[11] Some details are given in the Appendix.

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Appendix

1. Let us consider the derivation of Eq. (35) in more detail. We start from Eq. (33), which can be rewritten in the form:

$$a_1^R(k^2) = -\frac{e^2}{(2\pi)^3}4\pi(m^2+k^2)\frac{\partial}{\partial m^2}\int_0^1 dx\int_0^l p^2\,dp\left(p^2+m^2+\frac{k^2}{4}(1-x^2)\right)^{-1/2}$$

$$= -\frac{e^2}{(2\pi)^2}\sqrt{\frac{4}{k^2}}\int_0^l p^2\,dp(m^2+k^2)\frac{\partial}{\partial m^2}\int_0^1 dx(\alpha+1-x^2)^{-1/2};\ \text{where } \alpha=\frac{4(p^2+m^2)}{k^2},$$

$$= -\frac{e^2}{(2\pi)^2}\sqrt{\frac{4}{k^2}}\int_0^l p^2\,dp(4m^2+k^2)\left(\frac{4}{k^2}\right)\frac{\partial}{\partial\alpha}\int_0^{\frac{1}{\sqrt{1+\alpha}}}\frac{dy}{(1-y^2)^{1/2}}$$

$$= -\left(\frac{e^2}{8\pi^2}\right)\int_0^l\frac{p^2\,dp}{E}\frac{4m^2+k^2}{k^2+4m^2+p^2} \Rightarrow \text{after renormalization, see Eq.(34)}$$

$$= -\frac{e^2}{16\pi^2}(k^2)^2\int_{4m^2}^\infty\frac{dz^2\left(1-\frac{4m^2}{z^2}\right)^{3/2}}{z^2(z^2+k^2)}.$$

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