



FINITE ELEMENT MODELING OF REINFORCED CONCRETE BEAMS STRENGTHENED WITH CARBON FIBER COMPOSITES

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ABSTRACT: This paper presents the results of a comparison between a finite element model and experimental data of reinforced concrete beams strengthened with carbon fiber reinforced plastics. For the experimental program, simply supported beams, with 12 x 20 cm cross section and 2.25 m long span were tested. These beams have been submitted to short-term static loading tests from which it has been possible to evaluate the stresses and strains, the displacements, the crack widths and the failure mode. Strains gauges glued to the composite surface allowed the analysis of the behavior at the interface. The tensile and shear stresses at interface could be estimated. The numerical model consisted of a nonlinear finite element model for the flexure-shear response. The concrete was represented through plane stress isoparametric, eight nodes, finite elements. The concrete two-dimensional constitutive law was based on the orthotropic model proposed by Darwin. The concept of uniaxial equivalent strain and the two-dimensional failure criterion of Kupfer and Gerstle were adopted. The reinforcement was represented through an embedded model. Each steel bar was considered as a more rigid line inside of the concrete element, which just resists to axial efforts. The model includes a special interface element to simulate the bond between concrete and the external composite plate which is represented by truss elements.

1. INTRODUCTION

Every day more concrete structures have been presenting the need of rehabilitation and repair due to overload problems, use change, cracking, reinforcement bars corrosion, concrete deterioration, etc. Then, retrofit and repair systems are searched as a necessary alternative. Various materials and techniques are today available for the repair design and it is up to the engineer to choose the most appropriated for the case. The method of strengthening concrete structures with externally epoxy-bonded carbon fiber reinforced plastics (CFRP) is having an increasingly utilization. It offers the advantages of high strength, light weight, immunity to corrosion and efficiency of application. In this study, the Freyssinet System of CFRP for structural repair is evaluated. This is a composite made of a carbon fiber woven fabric impregnated with an epoxy resin. The woven fabric is bidirectional, with 70% of the carbon fibers in the principal direction and 30% in the transversal direction. The numerically obtained results by a finite element model are compared with the experimental data from tests carried out by Souza and Appleton [1, 2]. The

reinforced concrete beams - F1 and F5 - were tested with concentrated loads, accordingly the scheme presented in Fig. 1. The F2 beam was tested in such a way to simulate uniformly distributed loads, accordingly the scheme presented in Fig 2.

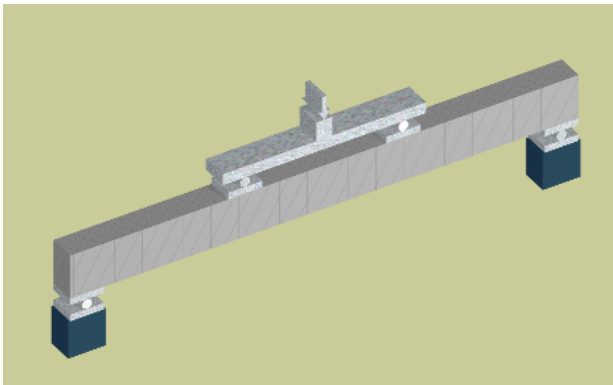


Figure 1 – Tests program scheme: F1 – F5 beams



Figure 2 – Tests program scheme: F2 beam

The reinforced concrete beams have a 12 x 20 cm cross section and a span 2.25 m long. Figure 3 shows the beams details. The beams characteristics and the materials properties are specified in Tables 1 and 2, respectively.

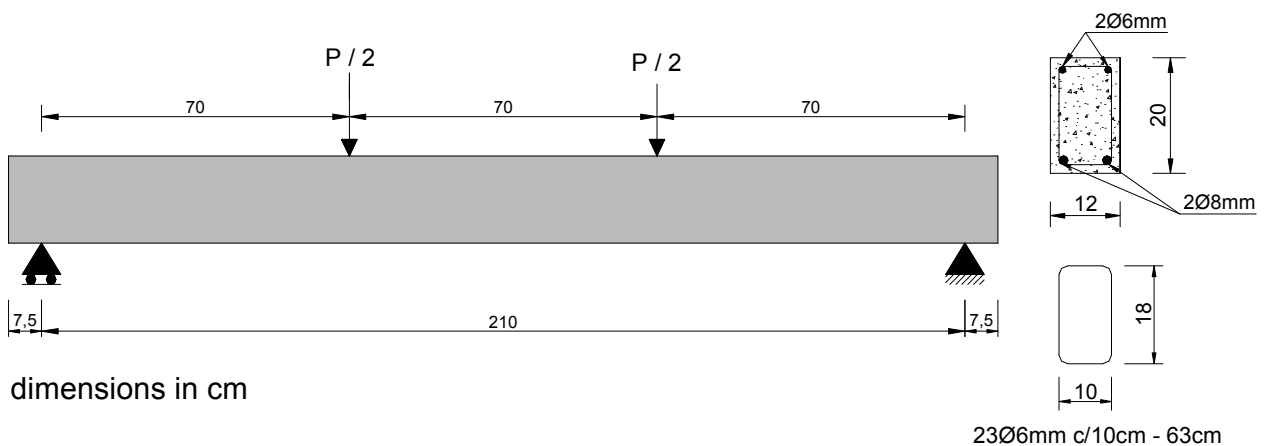


Figure 3 – Tested beams details

Table 1 – Tested beams characteristics

Beam	Applied CFRP composite	Loading type
F1	one composite layer at the bottom face	concentrated loads at the thirds
F2	one composite layer at the bottom face	uniformly distributed load
F5	two composite layers at the bottom face	concentrated loads at the thirds

According to the Freyssinet manufacturer's indications [3], the physical and mechanical properties of the carbon fiber composite are:

- Composite thickness: 0.43 mm;
- Composite width: 75 mm;
- Ultimate bond stress: 1.50 MPa;
- Epoxy resin tensile strength: 29.30 MPa;
- Tensile rupture strain: 1.37%;
- Tensile rupture stress: 1400 MPa;
- Tensile Young's modulus: 105,000 MPa.

The mechanical characteristics of the materials were tested according to the normalized standards. The obtained values of these tests are presented at Table 2.

Table 2 – Materials properties

Material	Property	Description	
Concrete	$f_{cm} = 33 \text{ MPa}$	Mean compressive strength	
	$f_{ctm} = 3.10 \text{ MPa}$	Mean tensile strength	
	$E_{cm} = 30,500 \text{ MPa}$	Mean Young's modulus	
Steel	8 mm bar	$f_{sy} = 486 \text{ MPa}$	Yielding stress
		$E_s = 210,000 \text{ MPa}$	Mean Young's modulus
	6mm bar	$f_{sy} = 557 \text{ MPa}$	Yielding stress
		$E_s = 210,000 \text{ MPa}$	Mean Young's modulus
Composite	$\sigma_{fu} = 1425 \text{ MPa}$	Tensile rupture stress	
	$\epsilon_{fu} = 0.66 \%$	Tensile rupture strain	
	$E_f = 138,000 \text{ MPa}$	Tensile Young's modulus	

2. FINITE ELEMENT MODEL

The concrete is represented by two-dimensional, eight node, isoparametric, plane stress finite elements in the theoretical analysis. The concrete biaxial constitutive model involves the uniaxial equivalent strain concept of Darwin [4] and the biaxial failure criteria for concrete of Kupfer and Gerstle [5]. The model for the tensile behavior of concrete includes a descending stress-strain curve branch after cracking to incorporate the tension-stiffening effect.

The steel reinforcement is represented by an embedded model, based on the work of Elwi and Hrudey [6]. Each steel bar is considered as a more rigid material line inside the concrete element which resists only the axial force in the bar direction. Perfect adherence is assumed between the steel reinforcement and the concrete that involves it. In this way, the steel reinforcement stiffness matrix has the same dimensions as the concrete element stiffness matrix. The steel stress-strain relationship is bilinear.

A three-node truss element is used to model the CFRP composite. Due to efforts transference between the composite and the concrete, bond stresses appear at the interface of these two materials. These bond stresses may cause the premature debonding of the composite leading to the structure failure. A six node interface element, with quadratic shape functions, is employed for the determination of the bond stresses. This interface element is based on the formulation presented by Adhikary and Mutsuyoshi [7], according to Fig. 4.

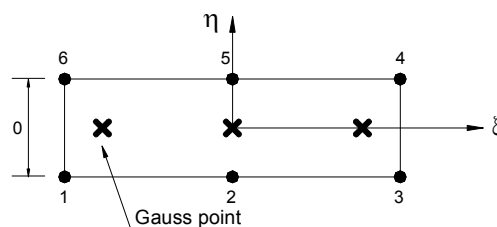


Figure 4 – Interface finite element

The slip “s” between the composite and concrete can be evaluated using the following equation

$$s = N_1 (u_6 - u_1) + N_2 (u_5 - u_2) + N_3 (u_4 - u_3) , \quad (1)$$

where $N_i(\xi)$ are the shape functions given in reference [7] and u_i are the nodal displacements of the interface element in the horizontal direction.

The bond stress (τ) between concrete and the CFRP composite can be evaluated as a function of the relative slip “s” according to the CEB-FIP 1990 Model Code [8] bond-slip law, which is given by the following equations

$$\tau = \tau_{\max} \left(\frac{s}{s_1} \right)^\alpha \quad \text{for } 0 \leq s \leq s_1 \quad (2)$$

$$\tau = \tau_{\max} \quad \text{for } s_1 < s \leq s_2 \quad (3)$$

$$\tau = \tau_{\max} - (\tau_{\max} - \tau_f) \left(\frac{s - s_2}{s_3 - s_2} \right) \quad \text{for } s_2 < s \leq s_3 \quad (4)$$

$$\tau = \tau_f \quad \text{for } s_3 < s \quad (5)$$

The adopted parameters for the evaluation of the bond stress between the concrete and the CFRP composite are given in Table 3, according Silva [9] and Aurich [10].

Table 3 – Parameters for the bond stress evaluation

S_1 (mm)	S_2 (mm)	S_3 (mm)	α	τ_{\max} (MPa)	τ_f (MPa)
0.08	0.08	0.65	0.6	1.5	$0.1\tau_{\max}$

3. FINITE ELEMENT DISCRETIZATION

All the analyses were performed using a mesh of $6 \times 2 = 12$ elements for concrete, 6 elements for the interface and 12 truss elements for the CFRP composite. Due to geometry and loading symmetry only half of the beam was analyzed. The mesh employed in the analyses is showed in Fig. 5. The loading showed in this figure is for F1 and F5 beams, while in the F2 beam, the loading is uniformly distributed.

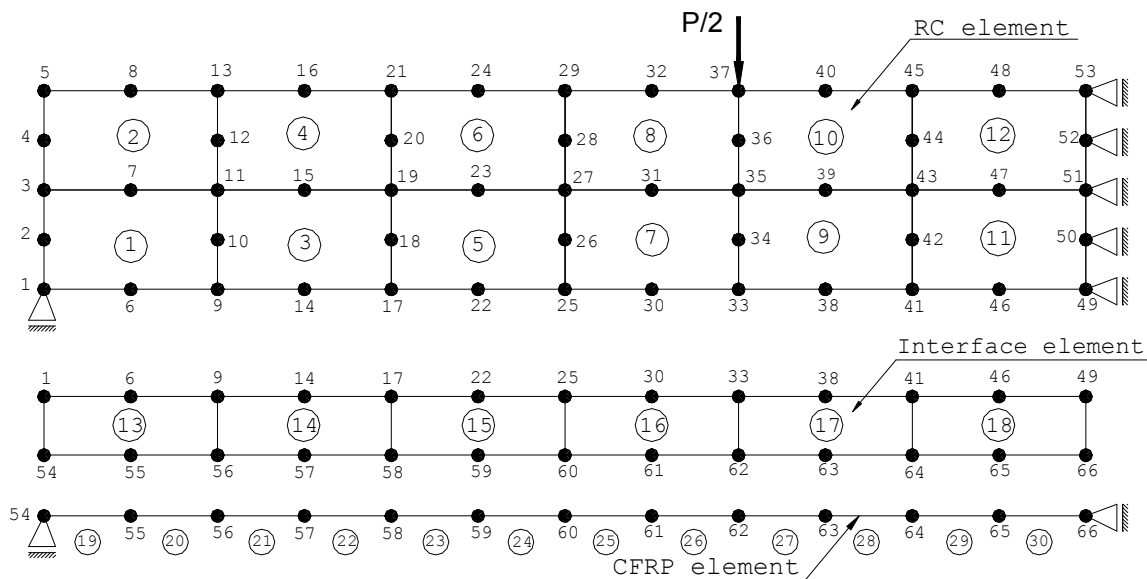


Figure 5 – Finite elements mesh

4. RESULTS AND DISCUSSIONS

In the tests, it was observed a sudden failure of the beams when the composite suffered debonding and/or rupture. The composite debonding or rupture occurred always under a bending crack with a large opening.

The numerical and experimental results for beam F1 are compared in Figs. 6 and 7. Figure 6 presents the evolution of the midspan deflection with the increasing load, while Fig. 7 shows the composite strains at the same section. A loading-deflection curve corresponding to the hypothesis of perfect bond between the composite and the concrete beam was added to Fig. 6. Naturally, when perfect bond was assumed, it was obtained a greater stiffness than when the slip between the composite and concrete was considered.

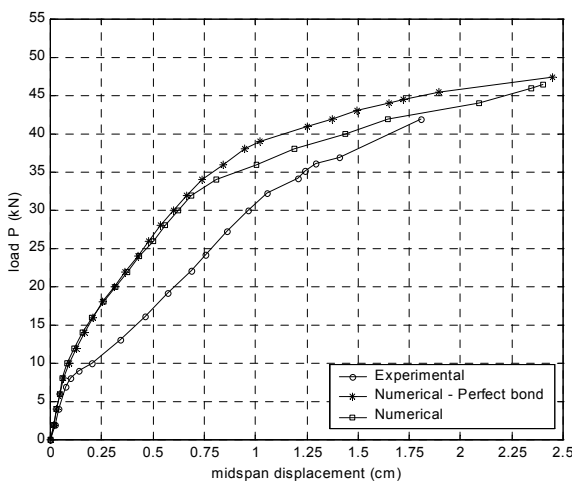


Figure 6 – Load - deflection curve: beam F1

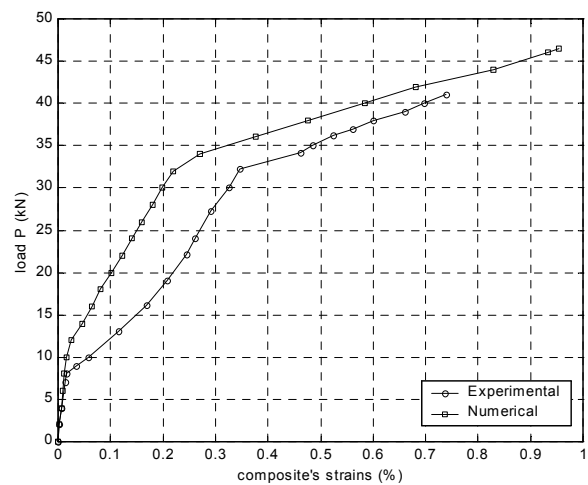


Figure 7 – Composite strains: beam F1

The numerically obtained bond stresses between the CFRP composite and the concrete beam are shown in Fig. 8, for various load levels. The bond stress peaks occurred in a zone of intense cracking. It was observed experimentally that the composite debonding took place in this same zone. The bond stresses obtained in the numerical analysis are compared with those achieved in the tests in Fig. 9, for two load levels.

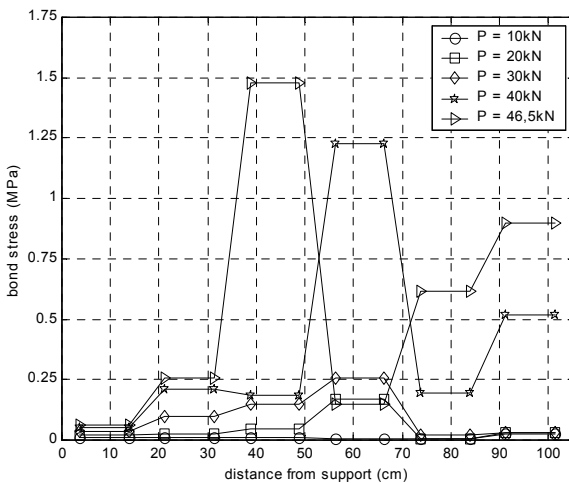


Figure 8 – Bond stresses: beam F1

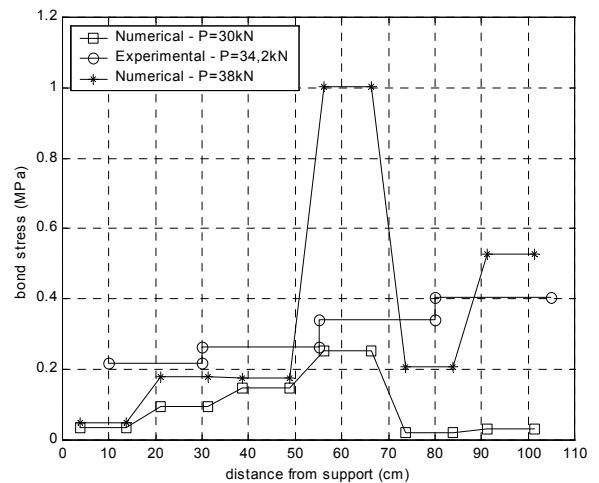


Figure 9 – Bond stresses along the composite: beam F1

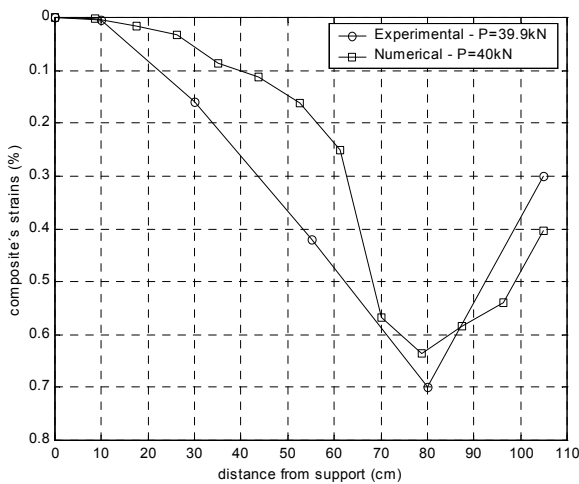


Figure 10 – Strains evolution along the composite: beam F1

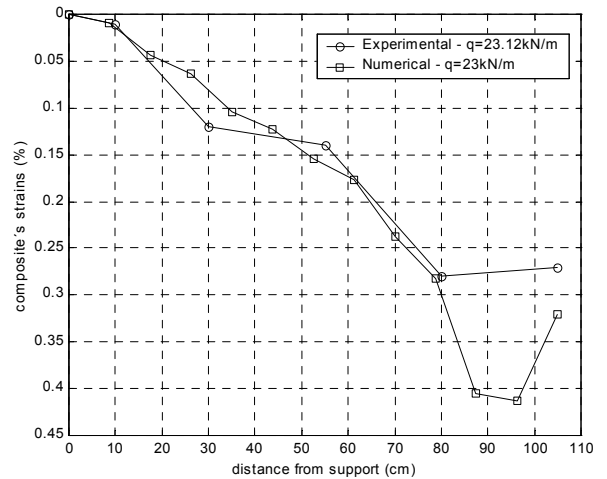


Figure 11 – Strains evolution along the composite: beam F2

The Figs. 10 and 11 present a comparison between experimental and numerical strains in the composite along F1 and F2 beams for one load level.

The numerical and experimental results of the greatest values of the deflections and strains at the composite for beam F2 are shown in Figs. 12 and 13, respectively. It must be noted that beam F2 is identical to beam F1; the only difference is that beam F2 has a uniformly distributed load.

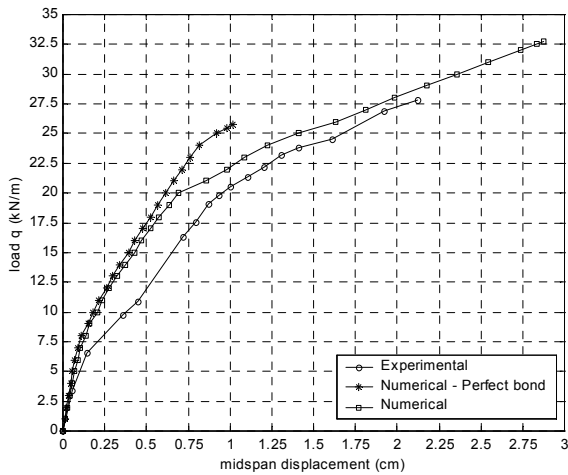


Figure 12 – Load - deflection curve: beam F2

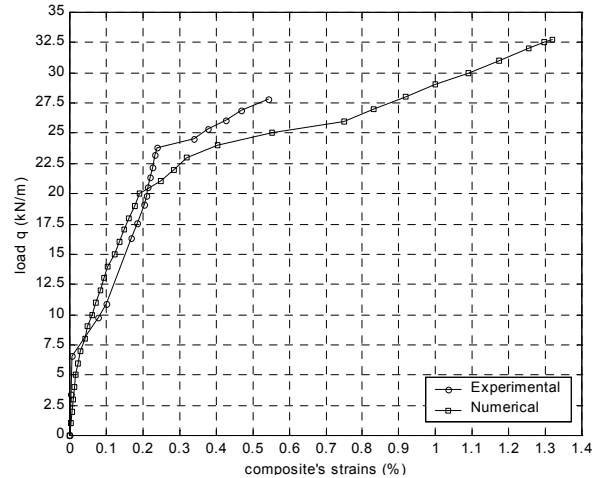


Figure 13 – Composite strains: beam F2

The calculated bond stresses at the interface between the concrete beam and the CFRP composite for various load levels are given in Fig. 14. It was again observed that the bond stresses peaks occurred in a zone of intense cracking.

Figure 15 compares the numerical and experimental values of the bond stresses for different load levels.

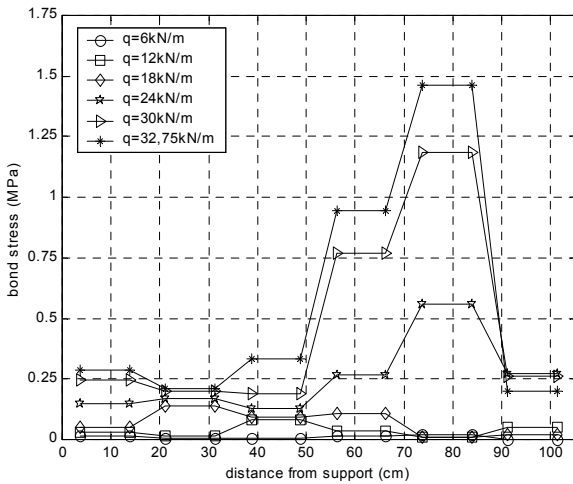


Figure 14 – Bond stresses: beam F2

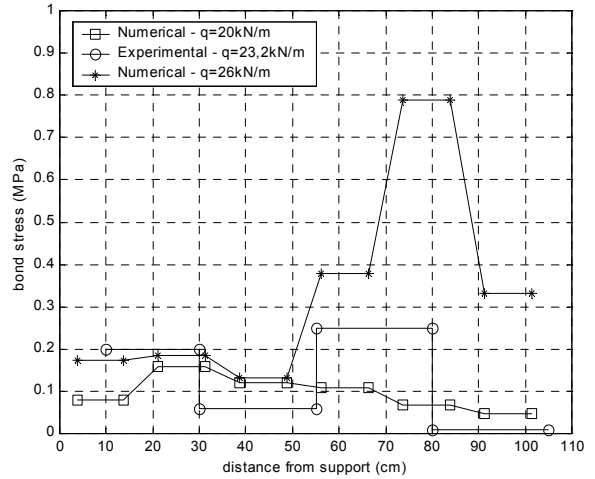


Figure 15 – Bond stresses along the composite: beam F2

The loading scheme of beam F5 was identical to that of beam F1. The only difference between the two beams was the number of composite layers. The greatest values of displacements and strains in the composite and the bond stresses are shown in Figs. 16 to 19. These results are similar to those observed in the aforementioned beams.

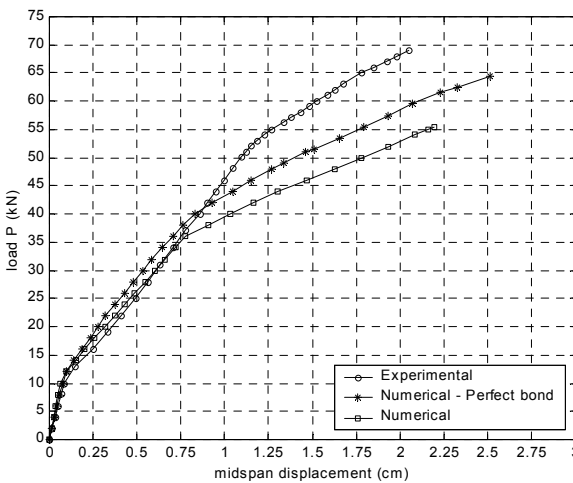


Figure 16 – Load - deflection curves: beam F5

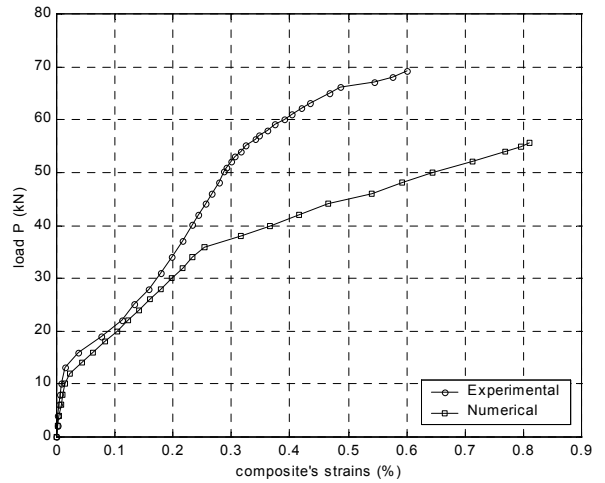


Figure 17 – Composite strains: beam F5

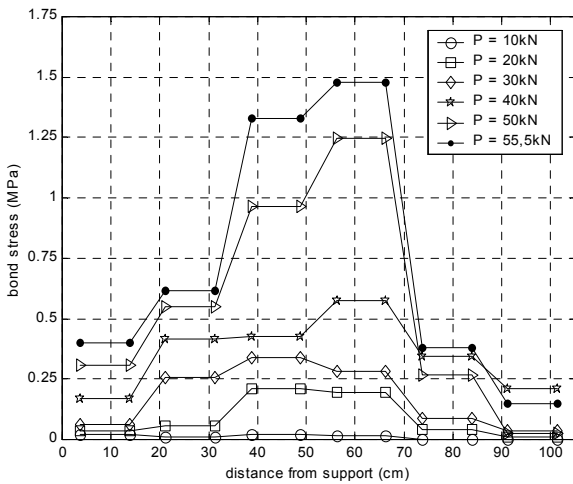


Figure 18 – Bond stresses: beam F5

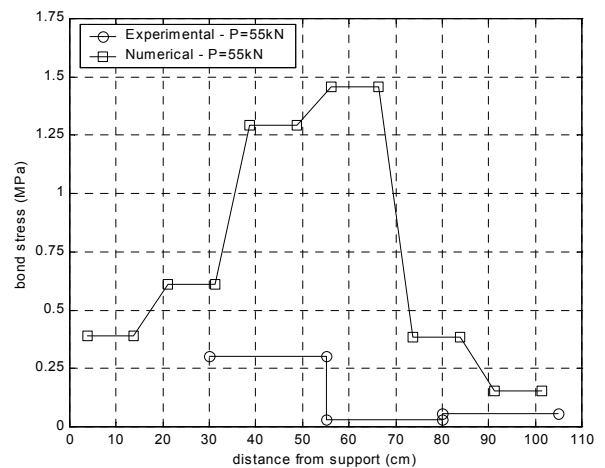


Figure 19 – Bond stresses along the composite: beam F5

5. CONCLUDING REMARKS

The most relevant aspect of this study is the possibility of a numerical prediction of the failure mode of CFRP composite strengthened beams by a finite element model. As experimentally verified, the rupture condition is reached in an intense cracking region, where the ultimate value of the bond stress between the concrete beam and the composite is attained.

The beams F1 and F2, with only one composite layer, achieved almost the same rupture bending moment, although they were submitted to different loading schemes. However, it was verified that the bond stresses distribution between the concrete beam and the composite were different in these two beams. From Fig. 8, it can be observed that the bond stress peak moves towards the supports in the beam F1. This fact could not be noted in the beam F2.

By comparing the results obtained for the beams F1 and F5, it was noted that the addition of one more CFRP composite layer enhanced the beam loading capacity. Nevertheless, it must be observed that the bond stresses distribution was not modified. A rapid growth of the bond stresses was observed after the concrete tensile strength was surpassed.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

- [1] Souza RHF, Appleton JAS. Anchorage details and bond aspects of reinforced concrete beams strengthened with carbon fiber composites. *Composites in Construction*, Italy, 2003.
- [2] Souza RHF, Appleton JAS. Estudo experimental sobre o reforço de vigas de concreto armado com tecido compósito de fibras de carbono. *45º Congresso Brasileiro do Concreto*. 2003.
- [3] Freyssinet. *Cahier des clauses techniques – Renforcement du beton par collage de tissu de fibres de carbone procede TFC*. 1997, p. 85-107.
- [4] Darwin D, Pecknold D. Nonlinear biaxial stress-strain law for concrete. *Journal of Engineering Mechanics Division*, 1977;103;229:241.
- [5] Kupfer HB, Gerstle KH. Behavior of concrete under biaxial stresses. *Journal of Engineering Mechanics Division*, 1973; 99;853:866.
- [6] Elwi AE, Hrudey TM. Finite Element model for curved embedded reinforcement. *Journal of Engineering Mechanics Division*, 1989; 115;740:745.
- [7] Adhikary BB, Mutsuyoshi H. Numerical simulation of steel-plate strengthened concrete beam by a nonlinear finite element method model. *Construction and Building Materials*, 2002; 16; 291:301.
- [8] Comité Euro-International du Béton. CEB-FIP Model Code 1990. *Design Code*. London: Thomas Telford Services; 1993.
- [9] Silva PASC. *Modelação e análise de estruturas de betão reforçadas com FRP*. Faculdade de Engenharia da Universidade do Porto. 254p. MSc. Thesis; 1999.
- [10] Aurich M. *Modelo da ligação entre concreto e armadura na análise de estruturas de concreto pelo método dos elementos finitos*. Universidade Federal do Rio Grande do Sul. 115p. MSc. Thesis; 2001.