# Hand Gesture Recognition in an Interval Fuzzy Approach 

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#### Abstract

This paper introduces an interval fuzzy rule-based method for the recognition of hand gestures acquired from a data glove, with an application to the recognition of hand gestures of the Brazilian Sign Language. To deal with the uncertainties in the data provided by the data glove, an approach based on interval fuzzy logic is used. The method uses the set of angles of finger joints for the classification of hand configurations, and classifications of segments of hand gestures for recognizing gestures. The segmentation of gestures is based on the concept of monotonic gesture segment. Each gesture is characterized by its list of monotonic segments. The set of all lists of segments of a given set of gestures determine a set of finite automata able to recognize such gestures.


## 1. Introduction

There is an extensive literature about methods and systems for gesture recognition $[1,2,6,10,11,13,18,19,21]$ in general, and hand gesture recognition [7] in particular, such as, e.g.: systems for the recognition of 3-D and 2-D gestures captured by different devices (data gloves, cameras etc.) [6], methods based on fuzzy logic and fuzzy sets $[2,4,6,21]$, neural networks [11], hidden Markov models [19, 18], hybrid neuro-fuzzy methods $[1,7]$, etc. In particular, considering sign language recognition, some literature can be found related to fuzzy methods, such as, e.g, fuzzy decision trees [10] and neuro-fuzzy systems [1].

In this paper, we propose an interval fuzzy rule-based method for the recognition of hand gestures acquired from a data glove, extending the work presented in [4] to consider the uncertainties in the data provided by the glove. We apply the method to the recognition of hand gestures of LIBRAS, the Brazilian Sign Language [8].

The method uses the set of angles of finger joints (given as intervals that enclose the uncertainties of the data obtained by the glove sensors) for the classification of hand configurations, and classifications of sequences of hand configurations for

[^0]recognizing gestures. The segmentation of gestures is based on the concept of monotonic gesture segment, sequences of gestures in which the variations of the angles of the finger joints have the same sign (non-increasing or non-decreasing). Each gesture is characterized by a list of monotonic segments, which determine a set of finite automata, which are able to recognize the gestures being considered.

The paper is organized as follows. Section 2. presents some concepts of fuzzy systems. The interval fuzzy approach is discussed in Section 3.. Our interval fuzzy rule-based method for hand gesture recognition is introduced in Section 4.. A case study is presented in Section 5.. Section 6. presents the Conclusion and related work.

## 2. Fuzzy Systems

Fuzzy set theory [23] is the oldest and most widely used theory for soft computing, which deals with the design of flexible information processing systems, with applications in control systems, decision making, expert systems etc.

A fuzzy system implements a function of $n$ variables, given by a linguistic description of the relationship between them. Figure 1 illustrates the architecture of standard fuzzy systems. The fuzzificator computes the membership degrees of the crisp input values to the linguistic terms (fuzzy sets) associated to each input linguistic variable. The rule base contains the inference rules that associate linguistic terms of input linguistic variables to linguistic terms of output linguistic values. The information manager is responsible for searching in the rule base which rules are applicable for the current input. The inference machine determines the membership degrees of the output values in the output sets, by the application of the rules selected in the rule base. The defuzzificator gives a single output value as a function of the output values and their membership degrees to the output sets.


Figure 1: A standard fuzzy systems

However, many approximate methods do not produce a single final result, presenting several solutions to a single problem (e.g., the different classes to which a given input may belong), as, e.g., in fuzzy rule-based methods for pattern recognition [16]. Interval fuzzy rule-based systems consists of a generalization of such systems, by considering interval data type and interval membership degree values.

## 3. Interval Mathematics: some important concepts

Interval Mathematics was introduced in [17] for the automatic and rigorous controlling of the errors that arise in numerical computations, providing techniques to deal with the uncertainty and to obtain verified results in several different contexts (see, e.g., $[3,9,14]$ ). Any real number $x \in \mathbb{R}$ that is uncertain for some reason (e.g., if it is obtained by a measuring instrument with limited resolution) is represented by a real interval $X=\left[x_{1} ; x_{2}\right]$, with $x_{1}, x_{2} \in \mathbb{R}$ and $x_{1} \leq x \leq x_{2}$. The set of intervals is denoted by $\mathbb{I R} . x_{1}$ and $x_{2}$ denote, respectively, the left and right endpoints of $X$.

A machine interval has floating point numbers as endpoints and outward roundings are used to guarantee that the resulting output interval of any computation process contains the actual result, with the range of the output interval being the indicative of the maximum error occurred in the process. The arithmetical operations $*_{\mathbb{R} \mathbb{R}} \in\{+,-, \times, \div\}$ are defined, for all $X, Y \in \mathbb{\mathbb { R }}$, as $X *_{\mathbb{R}} Y=\{x * y \mid x \in X, y \in Y\}$. For $X=\left[x_{1}, x_{2}\right], Y=\left[y_{1}, y_{2}\right] \in \mathbb{R}$, they are explicitly given by [17]:

$$
\begin{gather*}
X+Y=\left[x_{1}+y_{1}, x_{2}+y_{2}\right] ; \quad X-Y=\left[x_{1}-y_{2}, x_{2}-y_{1}\right]  \tag{3.1}\\
X \times Y=[\min \rho, \max \rho], \text { with } \rho=\left\{x_{1} y_{1}, x_{1} y_{2}, x_{2} y_{1}, x_{2} y_{2}\right\} ;  \tag{3.2}\\
X \div Y=X \times\left[y_{2}^{-1}, y_{1}^{-1}\right], \text { if } 0 \notin Y . \tag{3.3}
\end{gather*}
$$

For the purpose of this work, a sign of $X=\left[x_{1} ; x_{2}\right] \in \mathbb{I} \mathbb{R}$ is defined as:

$$
\operatorname{sign}\left(\left[x_{1}, x_{2}\right]\right)= \begin{cases}+ & \text { if } x_{1} \geq 0 \text { and } x_{2}>0  \tag{3.4}\\ - & \text { if } x_{1}<0 \text { and } x_{2} \leq 0 \\ 0 & \text { otherwise }\end{cases}
$$

The range of a real function $f: \mathbb{R} \rightarrow \mathbb{R}$ over $X \in \mathbb{R}$ is given by $\bar{f}(X)=\{f(x) \mid$ $x \in X\}$. An interval representation of $f$ is an interval function $F: \mathbb{R} \rightarrow \mathbb{R}$ such that, for each $X \in \mathbb{R}, x \in X, f(x) \in F(x)$ [20]. Although there is not a unique interval representation for a given real function, it always holds that $\bar{f}(X) \subseteq F(X)$. Observe that it is not always possible to represent interval functions in the cartesian plan. However, in some cases, an interval function $F(X)$ can be represented by an interval of real functions $F(X)=[\inf \{f(x) \mid x \in X\} ; \sup \{g(x) \mid x \in X\}]$, where $f$ and $g$ are real functions such that $f \leq g$, which is denoted by $[f, g]$. In this work, we consider that the membership degrees are interval functions where $f=g$, denoted by $[f]$, which is the best interval representation of $f$, i.e., for any other interval representation $F$ of $f, \quad[f](X) \subseteq F(X)[20]$.

## 4. The Interval Fuzzy Rule-Based Method

We consider a hypothetical data glove with 19 sensors, located at joints and separations between fingers, as shown in Fig. 2. The fingers are labelled as: F1 (little finger), F2 (ring finger), F3 (middle finger), F4 (index finger) and F5 (thumb). The joints in the fingers are labelled as J1 (the knuckle), J2 and J3, for each finger. A separation between two fingers Fi and Fj is labelled as Sij .

Since any movement can be represented as a sequence of frames, a hand movement using a data glove is represented as a sequence of hand configurations, one


Figure 2: Localization of sensors in the data glove
for each discrete time instant. That is, at each time instant, the data glove sensors should provide the set of angles of joints and finger separation that characterizes a hand configuration. These angles are represented as interval angles $[x-\epsilon ; x+\epsilon]$ that enclose the uncertainties in the processing, where $x$ is the angle given by a sensor and $\epsilon>0$ is the equipment tolerance, given by the manufacturer.

In order to simulate this data transfer, a random generator of hand configurations was implemented, generating at each instant one hand configuration represented by a tuple of interval angles corresponding to each sensor shown in Fig. 2:

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((F1J1,F1J2,F1J3), S12, (F2J1,F2J2,F2J3), S23, (F3J1,F3J2,F3J3),
    S34, (F4J1,F4J2,F4J3), S45, (F5J1,F5J2,F5J3) )
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Given a hand configuration $c$ and a sensor $s$, denote the interval value of each sensor angle by $s(c)$, e.g., F1J1(c), S45(c), etc.

### 4.1. Fuzzification

Considering that fuzzy systems usually work with totally ordered data, the designer often has difficulty to express the membership degrees as a function in the cartesian plan for data for which there is no natural total order. Notice that it is not uncommon to consider data which are obtained with an imprecision and therefore which are handled more appropriately with intervals, which have no natural order. Thus, interval-valued fuzzy logic [5] was introduced to deal with interval membership degrees, i.e., subintervals of $[0 ; 1]$ that allow the expression of the uncertainty of the expert about the classification of the data in linguistic terms.

In the case of this work, to each sensor corresponds a linguistic variable, whose values are linguistic terms representing typical angles of joints and separations. For the joints in the fingers (linguistic variables F1J1, F1J2, F1J3, etc.) the linguistic terms are: STRAIGHT (St), CURVED (Cv) and BENT (Bt). For the separations between fingers F1 and F2, F2 and F3, F4 and F5 (linguistic variable S12, S23, S45),
the linguistic terms are: CLOSED (Cl), SEMI-OPEN (SOp) and OPEN (Op). For the separations between fingers F3 and F4 (linguistic variable S34), the linguistic terms are: CROSSED (Cr), CLOSED (Cl), SEMI-OPEN (SOp) and OPEN (Op). Figures $3,4,5$ and 6 show the (interval) fuzzification for those variables.


Figure 3: Fuzzification of the linguistic variable of the joint F5J2 of the finger F5


Figure 4: Fuzzification of the linguistic variables of remaining finger joints


Figure 5: Fuzzification of the linguistic variable of the separation S45 between the index finger F4 and the thumb finger F5

### 4.2. The Interval Inference Process

The more generic and accepted way to consider fuzzy generalization of classical connectives are based on triangular norms (t-norms, t-conorms, fuzzy negations and fuzzy implications (residuum)) [15]. In [5] those concepts were defined in an


Figure 6: Fuzzification of the linguistic variables of the separations: (a) S34 between the middle finger F3 and the index finger F4, and (b) between remaining fingers
interval approach. In this paper, we use the generalization of the Gödel t-norm:

$$
\begin{equation*}
G([a, b],[c, d])=[\min \{a, c\} ; \min \{b, d\}] . \tag{4.1}
\end{equation*}
$$

For example, consider the (interval) membership degrees $\varphi$ for the joints of the index finger (shown in Fig. 4) and the rule

If F4J1 is STRAIGHT and F4J2 is CURVED and F4J3 is CURVED Then F4 is StCvCv
If the angles provided by the data glove for the joints 1,2 and 3 are $7^{\circ}, 15^{\circ}$ and $13^{o}$, respectively, and the tolerance ${ }^{4}$ is $\epsilon=1^{o}$, the membership degrees $\varphi$, extracted from Fig. 4, are the following: $\quad \varphi_{F 4 J 1}([6 ; 8])=\left[\frac{15-8}{8} ; 1\right]=[0.875 ; 1]$, $\varphi_{F 4 J 2}([14 ; 16])=\left[\frac{14-7}{8} ; 1\right]=[0.875 ; 1] \quad$ and $\quad \varphi_{F 4 J 3}([12 ; 14])=\left[\frac{12-7}{8} ; \frac{14-7}{8}\right]=$ [0.625; 0.875] . Then, using the interval t-norm $G$ given in (4.1), we have

$$
G(G([0.875 ; 1],[0.875 ; 1]),[0.625 ; 0.875])=[0.625 ; 0.875]
$$

meaning that $F 4$ is in StCvCv with interval degree [0.625;0.875]. If we use the product interval t-norm, i.e., $P([a ; b],[c ; d])=[a c ; b d]$, the interval membership degree of finger 4 to StCvCv would be [0.546;0.875].

### 4.3. The Recognition Process

The hand gesture recognition process is divided into four steps: (1) recognition of finger configurations; (2) recognition of hand configurations; (3) segmentation of the gesture in monotonic hand segments; (4) recognition of the sequence of monotonic hand segments. For the Step 1 (recognition of finger configurations), 27 possible finger configurations are considered, for each finger. These configurations are codified in the format XYZ, where $\mathrm{X}, \mathrm{Y}$ and Z are the values of the linguistic variables corresponding to the first joint J1, the second joint J2 and the third joint J3, respectively. For example, StStSt indicates that the three joints are STRAIGHT, StCvCv indicates that the first joint is STRAIGHT whereas the others are CURVED, etc.

[^1]The hand configuration is the main linguistic variable of the system, denoted by HC , whose linguistic terms are names of hand configurations, which names are application dependent. For instance, in Sect. 5., names of Brazilian Sign Language (LIBRAS) hand configurations (see Fig. 7) were used for such linguistic terms.

The 27 possible finger configurations determine 27 inference rules that calculate membership degree of each finger to each configuration. For example, see the rule for the index finger in the previous subsection.

Step 2 (recognition of hand configurations) determines the hand configuration, considering each finger configuration and separation between fingers. For example, the rule for the hand configuration [G] (Fig. 7) is:

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If F1 is BtBtSt and S12 is Cl and F2 is BtBtSt and S23 is Cl and F3 is BtBtSt
    and S34 is Cl and F4 is StStSt and S45 is Cl and F5 is StStSt
Then HC is [G]
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In Step 3 (segmentation of the gesture in monotonic hand segments), we divide each gesture in a sequence of $k$ limit hand configurations $l_{1}, \ldots, l_{k}$, where $l_{1}$ and $l_{k}$ are the initial and the terminal gesture configurations, respectively. The limit configurations are such that, for each sensor $s$ and $i=1, \ldots, k-1$, it holds that: (i) they are not coincident, i.e., $\quad\left|s\left(l_{i+1}\right)-s\left(l_{i}\right)\right| \leq 2 \epsilon$, where $\epsilon$ is the equipment tolerance; (ii) they are the limits of monotonic segments: (a) for each $c$ between $l_{i}$ and $l_{i+1}, \quad \operatorname{sign}\left(s(c)-s\left(l_{i}\right)\right)=\operatorname{sign}\left(s\left(l_{i+1}\right)-s\left(l_{i}\right)\right)$ and (b) for each $c^{\prime}$ after $l_{i+1}, \quad \operatorname{sign}\left(s\left(c^{\prime}\right)-s\left(l_{i+1}\right)\right) \neq \operatorname{sign}\left(s\left(l_{i+1}\right)-s\left(l_{i}\right)\right) \quad$ (a sign equal to 0 is compatible with both negative and positive signs), where the sign of an interval was given in (3.4). The limit hand configurations are the points that divide the gesture into monotonic segments, i.e., segments in which each sensor produces angle variations with constant (or null) sign. $l_{i}$ and $l_{i+1}$ are, respectively, the initial and the terminal hand configurations of a monotonic segment $l_{i} l_{i+1}$.

The procedure for step 3 is the following. To find any monotonic segment $l_{i} l_{i+1}$, the next $n$ configurations sent by the data glove after $l_{i}$ are discarded, until a configuration $c_{n+1}$, such that $\operatorname{sign}\left(s\left(c_{n+1}\right)-s\left(c_{n}\right)\right) \neq \operatorname{sign}\left(s\left(c_{n}\right)-s\left(l_{i}\right)\right) \quad\left(\right.$ or, $c_{n+1}$ is the last configuration of the gesture). Then, $c_{n}$ (resp., $c_{n+1}$ ) is the terminal hand configuration $l_{i+1}$ of the considered monotonic segment, and also coincides with the initial configuration of the next segment $l_{i+1} l_{i+2}$ (if there is one). The process starts with $l_{i}=l_{1}$, which is the initial gesture configuration, and is repeated until the end of the gesture, generating the list of $k$ limit hand configurations.

In Step 4 (recognition of the sequence of monotonic hand segments), the recognition of each monotonic segment $l_{i} l_{i+1}$ is performed using a list of reference hand configurations $r_{1}, r_{2}, \ldots, r_{m}$ that characterizes the segment, where $r_{1}$ and $r_{m}$ are the initial and terminal hand configurations of the segment, respectively.

A monotonic segment is recognized by checking that it contains its list of reference hand configurations. The process is equivalent to a recognition based on a linear finite automaton (shown in Fig. 8), where $l_{i}=r_{1}$ and $l_{i+1}=r_{m}$.

## 5. Case Study: Hand Gestures of LIBRAS

As any other sign language, LIBRAS (Língua Brasileira de Sinais - Brazilian Sign Language) is a natural language endowed with all the complexity normally found
in the oral-auditive languages. In the various works on automatic recognition of sign languages that have been developed along the years (see Sect. 1.) the recognition of hand gestures has occupied a prominent place. To support that recognition process, a reference set of hand configurations is usually adopted, driven either from the linguistic literature on sign languages, or dynamically developed by the experimenters with an ad hoc purpose. For our purposes, we have chosen a standard set of hand configurations (some of them shown in Fig. 7), taken from the linguistic literature on LIBRAS [8]. Then, our method requires that each sign be thoroughly characterized in terms of its monotonic segments and the sequences of hand configurations that constitute such segments, and that the identification of the monotonic segments and hand configurations be manually provided to the system. Although a capture device such as a data glove can be used to help to identify the typical values of the angles of the finger joints, the final decision about the form of the membership functions that characterize the linguistic terms has to be explicitly taken and manually transferred to the system.


Figure 7: Some LIBRAS hand configurations
We illustrate the application of the method by the definition of the necessary parameters for the recognition of hand gestures that constitute the signs CURIOUS in LIBRAS. CURIOUS is a sign performed with a single hand placed right in front of the dominant eye of the signer, with the palm up and hand pointing forward. The initial hand configuration is the one named [G1] in Fig. 7. The gesture consists of the monotonic movement necessary to perform the transition from [G1] to $[\mathrm{X}]$
and back to [G1] again, such movements been repeated a few times (usually two or three). A possible analysis of the gestures that constitute the sign CURIOUS is:
Initial configuration: [G1];
Monotonic segment S1: [G1]-[G1X]-[X]; Monotonic segment S2: [X]-[G1X]-[G1];
State transition function for the recognition automaton: see Fig. 9.
To support the recognition of the monotonic segments of CURIOUS, we have chosen to use one single intermediate hand configuration, [G1X], which does not belong to the reference set (Fig. 7) and whose characterization in terms of the set of membership functions for linguistic terms was defined in an ad hoc fashion, for the purpose of the recognition of CURIOUS. Together with [G1] and [X], it should be added to the list of hand configurations used by the recognition system.


Figure 8: Automaton for the recognition Figure 9: Automaton for the recognition of monotonic segments of hand gestures of the sign CURIOUS.

## 6. Conclusion and Final Remarks

This paper presented a fuzzy rule-based for the recognition of hand gestures. The method is highly dependent on a detailed previous analysis of the features of the gestures to be recognized, and on the manual transfer of the results of that analysis to the recognition system. This makes it suitable for the application to the recognition of hand gestures of sign languages, because of the extensive analysis that linguists that have already done of those languages.

Unlike general gestures, sign language is highly structured so that it provides an appealing test bed for new ideas and algorithms before they are applied to gesture recognition. The recognition methods usually are mainly concerned with the difficulty presented by a large vocabulary sign language (as, e.g., in [10]) and how to reduce the recognition time without a great loss of accuracy (as, e.g., in [19]). Fuzzy methods, however, are worried mainly with representing the uncertainties involved in the process (as, e.g., in [22]), in other to increase the the quality of the results. The innovation of our method is the use of the association of fuzzy logic to interval mathematics to deal also with the imprecision of data provided by the data glove, which are then represented by real intervals. To avoid the difficulty in expressing the membership degrees for interval-valued data, we use interval membership degrees, i.e., subintervals of $[0 ; 1]$ that allow the expression of the uncertainty of the expert about the classification of the data in linguistic terms. Also, we remark the contribution of this work to the automatic recognition of LIBRAS, which has been rarely studied. Prototypes (available on request) of a random gesture generator and of the gesture recognizer were implemented in the programming language Python, using the module PyInterval for Interval Mathematics [12].

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Resumo. Introduz-se um método baseado em regras fuzzy para reconhecimento de gestos de mão através de uma luva de dados, com uma aplicação no reconhecimento de gestos de mãos da Língua Brasileira de Sinais. Para tratar das incertezas nos dados, adotou-se uma abordagem baseada em lógica fuzzy intervalar. O método usa o conjunto de ângulos das juntas dos dedos para a classificação de configurações de mão, e classificação de segmentos de gestos de mão para o reconhecimento de gestos. A segmentação dos gestos é baseada no conceito de segmentos de gestos monotônicos. Cada gesto é caracterizado por sua lista de segmentos monotônicos, que determina um autômatos finito capaz de reconhecer tal gesto.

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[^1]:    ${ }^{4}$ In general, the tolerance is provided by the equipment manufacturer.

