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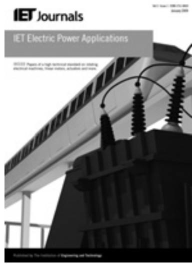

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Discrete-time sliding mode speed observer for sensorless control of induction motor drives

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Abstract: This study investigates the rotor speed estimation problem for induction motor drives. The authors propose the design of a scheme based on a discrete-time sliding mode observer which provides the rotor speed estimative. A new algorithm for discrete-time rotor speed estimation is developed and analysed. The conditions for the existence of a discrete-time sliding switching hyperplane are analysed. Moreover, conventional algorithms aiming at the chattering reduction and high-frequency switching on discrete-time implementation are discussed for use with the proposed technique. The stability analysis and parameter convergence of the proposed method are investigated for discrete-time solution. The algorithms developed are tested by experimental results based on fixed-point digital signal processor (DSP) platform (TMS320F2812). Therefore the results demonstrate the good performance of the proposed scheme.

1 Introduction

Controlled induction motor (IM) drives have been developed by several researchers in the last few decades. The most popular techniques are based on the field-oriented control schemes, where it is necessary to have the knowledge of IM state variables such as rotor/stator flux vector, rotor speed or electromagnetic torque. The measurement of these state variables is an onerous and hard task, which resulted in the development of state observers. Thus, the speed controlled IM drives without mechanical sensor use information from a rotor speed observer. The basic classes of IM drives without mechanical sensor schemes could be divided in the signal injection techniques and the model-based IM state equations methods [1].

Signal injection schemes operate successfully at a wide speed range including very low and zero stator frequencies; however, these schemes need an advanced design for the motor drive [2]. The schemes based on model of IM state equations are widely employed by industry and academic researchers mainly in the form of model reference adaptive system (MRAS) estimators, extend Kalman filter (EKF), recursive least square algorithms, neural and fuzzy methods or sliding-mode observers. The MRAS method is characterised by the use of two models. One of the models is dependent on the rotor speed, which is adjusted to minimise the error from the outputs of the two models [3–8]. The EKF method provides a recursive optimum stochastic state estimator [2, 9, 10].

In addition, sliding-mode control techniques have been extensively used in motor control drives, as has been described in [11–15]. These techniques are characterised by a strong robustness, disturbance rejection and simplicity implementation including the state observers designed for sensorless induction motor drives [16–21]. The paper [19] proposes a speed and a rotor time constant observer for induction machines, which eliminates the need of rotor flux information for the reconstruction of the rotor speed. Moreover, it presents the Lyapunov stability analysis of the method. In [20, 21], a continuous approach of stator current and rotor flux observers for IMs is proposed. From these estimations, an algorithm is carried out to calculate the rotor speed and the rotor resistance of the IM. In these papers the switching surfaces are defined by the errors of stator currents. On the other hand, in [13], the authors present a continuous flux and rotor speed observer where the functions of sliding mode are obtained from the observed vector flux and the vector stator current error. However, in the digital implementation, the design methodology of the analogue continuous-time sliding mode cannot be directly extended to discrete-time cases. The sampling time of the processor may bring chattering phenomenon to the designed sliding mode system owing to the fact that the switching frequency is limited to the sampling rate, such as reported in [22, 23].

Important contributions in the direction of discrete-time sliding mode methods have been reported in literature [24–28]. With the advent of powerful digital processors, the development of discrete-time techniques would help application designers to incorporate new features on machine drives and hardwares quickly. In this paper we propose a discrete-time sliding mode approach for rotor speed estimation of induction machines based on the studies of [19–21]. Here, the main contribution is the formulation of a theorem which gives the estimative of rotor speed. This new estimation algorithm is obtained and analysed in discrete-time form. Differently from previous papers that are related with IM sliding mode observers, in this study the state equations of the IM model are discretised and a discrete-time sliding mode observer is obtained. This method was chosen owing to the possibility of a direct implementation in digital signal processors without the discretisation of differential state equations. Using this approach it is possible to find the relationship between the time sampling and the observer gain to ensure the system stability. In addition, the conditions for the discrete-time sliding mode switching hyperplane existence are discussed. The upper and the lower bounds of the sliding mode gains are presented related with the IM parameters, sampling time and current estimation error. Conventional algorithms for the chattering reduction, such as sigmoid function and adaptive switching gain are employed. These algorithms are analysed based on the conditions for the discrete-time sliding switching hyperplane existence. Experimental results are presented to validate the proposed rotor speed observer and they demonstrated the effectiveness of the technique. These results show that the proposed scheme is simple and has good reference tracking and robustness capability.

This paper is organised as follows: Section 2 presents the mathematical model of the IM; Section 3 gives the discretised sliding mode current observer; Section 4 describes the proposed discrete-time rotor speed estimation algorithm; Section 5 shows the experimental results and algorithms for chattering reduction using the proposed scheme; and, Section 6 summarises the main topics presented in this paper.

2 IM model

The IM dynamics can be represented in the stationary reference frame ($\alpha\beta$) by continuous-time differential equations of stator currents and rotor flux [29], thus, for a three-phase N pole pair

$$\frac{\mathrm{d}}{\mathrm{d}t} i_{sq} = -\left(\frac{R_s}{\sigma L_s} + \beta\eta L_m\right) i_{sq} + \beta\eta\phi_{rq} - \beta N \omega_r \phi_{rd} + \frac{1}{\sigma L_s} v_{sq} \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} i_{sd} = -\left(\frac{R_s}{\sigma L_s} + \beta\eta L_m\right) i_{sd} + \beta N \omega_r \phi_{rq} + \beta\eta\phi_{rd} + \frac{1}{\sigma L_s} v_{sd} \quad (2)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \phi_{rq} = -\frac{R_r}{L_r}\phi_{rq} + N\omega_r\phi_{rd} + \frac{R_r}{L_r} L_m i_{sq} \quad (3)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \phi_{rd} = -\frac{R_r}{L_r}\phi_{rd} - N\omega_r\phi_{rq} + \frac{R_r}{L_r} L_m i_{sd} \quad (4)$$

$$T_e = \frac{3}{2}\frac{L_m}{L_r} N(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) \quad (5)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_r = -\frac{B_n}{J}\omega_r + \frac{1}{J}T_e - \frac{1}{J}T_L \quad (6)$$

where R_s, R_r, L_s, L_r, and L_m are the stator and the rotor resistances, stator, rotor and mutual inductances, respectively; i_{sq}, i_{sd}, ϕ_{rq}, ϕ_{rd}, v_{sq} and v_{sd} are the stator currents, the rotor flux and the stator voltages, ω_r is the rotor speed, T_e is the electromagnetic torque, T_L is the load torque, J is the moment of inertia and B_n is the friction coefficient. The constants in the equations above are defined as

$$\sigma \triangleq 1 - \frac{L_m^2}{L_s L_r}, \quad \beta \triangleq \frac{L_m}{\sigma L_s L_r}, \quad \eta \triangleq \frac{R_r}{L_r}$$

The transformed variables related to three-phase (ABC) system are given by

$$\boldsymbol{x}_{qd} = \boldsymbol{T}\boldsymbol{x}_{ABC} \quad (7)$$

where

$$\boldsymbol{T} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

3 Discretised sliding mode current observer

The aim of this paper was to develop the algorithm for direct implementation in digital signal processors, such as DSP and micro-controllers. Thus, the IM state equations (1) and (2) can be discretised by the Euler approximation method, choosing the sampling time T_s, as follows

$$i_{sq(k+1)} = \left(1 - \left(\frac{R_s}{\sigma L_s} + \beta\eta L_m\right)T_s\right) i_{sq(k)} + \beta\eta T_s \phi_{rq(k)} - \beta N\omega_{r(k)} T_s \phi_{rd(k)} + \frac{1}{\sigma L_s} T_s v_{sq(k)} \quad (8)$$

$$i_{sd(k+1)} = \left(1 - \left(\frac{R_s}{\sigma L_s} + \beta\eta L_m\right)T_s\right) i_{sd(k)} + \beta\eta T_s \phi_{rd(k)} + \beta N\omega_{r(k)} T_s \phi_{rq(k)} + \frac{1}{\sigma L_s} T_s v_{sd(k)} \quad (9)$$

From (8) and (9) a sliding mode current observer can be designed as

$$\hat{i}_{sq(k+1)} = \left(1 - \frac{R_s}{\sigma L_s}T_s\right)\hat{i}_{sq(k)} + \frac{1}{\sigma L_s} T_s v_{sq(k)} + V_{\alpha(k)} \quad (10)$$

$$\hat{i}_{sd(k+1)} = \left(1 - \frac{R_s}{\sigma L_s}T_s\right)\hat{i}_{sd(k)} + \frac{1}{\sigma L_s} T_s v_{sd(k)} + V_{\beta(k)} \quad (11)$$

where $\hat{i}_{sq(k+1)}$ and $\hat{i}_{sd(k+1)}$ are the estimated values of the stator currents, and $V_{\alpha(k)}$ and $V_{\beta(k)}$ are discontinuous functions of current errors, thus

$$V_{\alpha(k)} = -V_{0\alpha}\,\mathrm{sign}(s_{\alpha(k)}) = -V_{0\alpha}\,\mathrm{sign}(\hat{i}_{sq(k)} - i_{sq(k)}) \quad (12)$$

$$V_{\beta(k)} = -V_{0\beta}\,\mathrm{sign}(s_{\beta(k)}) = -V_{0\beta}\,\mathrm{sign}(\hat{i}_{sd(k)} - i_{sd(k)}) \quad (13)$$

where $V_{0\alpha}$ and $V_{0\beta}$ are positive gains to be designed and have upper and lower bounds as can be seen hereafter.

3.1 'Sliding conditions' for the discrete-time scheme

The conditions for the existence of the sliding switching hyperplane '$s_{i(k)}$' in discrete-time systems were developed and discussed in [27, 30–32]. From these studies it is

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possible to verify that the conditions derived from continuous-time systems are necessary but not sufficient in discrete-time systems; they do not ensure a stable convergence of the discrete-time sliding mode. The necessary and sufficient conditions can be obtained by using the Lyapunov function candidate $V_k = s^2_{i(k)}$, which results in

$$\Delta V_k = s^2_{i(k+1)} - s^2_{i(k)} = [s_{i(k+1)} + s_{i(k)}][s_{i(k+1)} - s_{i(k)}] \quad (14)$$

where the index 'i' represents the variables 'α' or 'β'.

Multiplying (14) by $\text{sign}^2[s_{i(k)}]$, it is possible to obtain the necessary and sufficient conditions for the existence of the hyperplane on $s_{i(k)}$ as presented in [27]

$$\begin{cases} [s_{i(k+1)} - s_{i(k)}]\,\text{sign}(s_{i(k)}) < 0 \\ [s_{i(k+1)} + s_{i(k)}]\,\text{sign}(s_{i(k)}) \ge 0 \end{cases} \quad (15)$$

From the global conditions for the existence of the discrete-time sliding switching hyperplane given in (15), it is possible to obtain the conditions for the existence of the $s_{\alpha(k)}$ and $s_{\beta(k)}$ hyperplanes. Considering the equations of estimation errors obtained from (8), (9), (10) and (11), it is possible to write the difference expressions of the estimation errors given by

$$s_{\alpha(k+1)} = \left(1 - \frac{R_s}{\sigma L_s} T_s\right) s_{\alpha(k)} - V_{0\alpha}\,\text{sign}(s_{\alpha(k)}) + T_s f_{\alpha(k)} \quad (16)$$

$$s_{\beta(k+1)} = \left(1 - \frac{R_s}{\sigma L_s} T_s\right) s_{\beta(k)} - V_{0\beta}\,\text{sign}(s_{\beta(k)}) + T_s f_{\beta(k)} \quad (17)$$

where $f_{\alpha(k)} \triangleq [\beta\eta L_m i_{sq(k)} - \beta\eta\phi_{rq(k))+\beta N\omega_{r(k)}\phi_{rd(k)}}]$ and $f_{\beta(k)} \triangleq [\beta\eta L_m i_{sd(k)} - \beta N\omega_{r(k)}\phi_{rq(k)} - \beta\eta\phi_{rd(k)}]$.

Aiming to obtain the first form of (15), it is possible to write

$$[s_{\alpha(k+1)} - s_{\alpha(k)}]\,\text{sign}(s_{\alpha(k)}) = -\frac{R_s}{\sigma L_s} T_s |s_{\alpha(k)}| - V_{0\alpha} + f_{\alpha(k)} T_s\,\text{sign}(s_{\alpha(k)}) \quad (18)$$

$$[s_{\beta(k+1)} - s_{\beta(k)}]\,\text{sign}(s_{\beta(k)}) = -\frac{R_s}{\sigma L_s} T_s |s_{\beta(k)}| - V_{0\beta} + f_{\beta(k)} T_s\,\text{sign}(s_{\beta(k)}) \quad (19)$$

From (18) and (19), it is possible to verify that the suitable choice of $V_{0\alpha}$ and $V_{0\beta}$ ensures the necessary condition for the existence of the hyperplane in $s_{\alpha(k)}$ and $s_{\beta(k)}$, which is given by

$$V_{0\alpha} > -\frac{R_s}{\sigma L_s} T_s |s_{\alpha(k)}| + f_{\alpha(k)} T_s\,\text{sign}(s_{\alpha(k)}) \quad (20)$$

$$V_{0\beta} > -\frac{R_s}{\sigma L_s} T_s |s_{\beta(k)}| + f_{\beta(k)} T_s\,\text{sign}(s_{\beta(k)}) \quad (21)$$

The sufficient condition, which is presented in the second term of (15), is ensured if the following conditions are guaranteed

$$V_{0\alpha} \le 2|s_{\alpha(k)}| + f_{\alpha(k)} T_s\,\text{sign}(s_{\alpha(k)}) - \frac{R_s}{\sigma L_s} T_s |s_{\alpha(k)}| \quad (22)$$

$$V_{0\beta} \le 2|s_{\beta(k)}| + f_{\beta(k)} T_s\,\text{sign}(s_{\beta(k)}) - \frac{R_s}{\sigma L_s} T_s |s_{\beta(k)}| \quad (23)$$

Inequalities (20) and (21) give the lower bounds for the gains $V_{0\alpha}$ and $V_{0\beta}$, which depend on the states and parameters of the machine. On the other hand, the upper bounds of $V_{0\alpha}$ and $V_{0\beta}$ are presented in (22) and (23), and are related with the parameters and states of the machine and with the sampling time and estimation error of the stator currents. From these equations it is possible to conclude that for a fixed time sampling the limits of $V_{0\alpha}$ and $V_{0\beta}$ change with the amplitude of the estimation error of the stator currents. Thus, it is possible to design adaptive gains that vary with the estimation error.

4 Rotor speed estimation algorithm

Assuming that the estimated stator currents (10) and (11) track the measured stator currents (8) and (9), the discontinuous functions of sliding mode $V_{\alpha(k)}$ and $V_{\beta(k)}$, which include the rotor speed variable, can be approximated by the following equations

$$\begin{aligned} V_{\alpha(k)} &\simeq L_{\alpha eq(k)} \\ &= \beta\eta T_s \phi_{rq(k)} - \beta N\omega_{r(k)} T_s \phi_{rd(k)} - \beta\eta L_m T_s i_{sq(k)} \end{aligned} \quad (24)$$

$$\begin{aligned} V_{\beta(k)} &\simeq L_{\beta eq(k)} \\ &= \beta N\omega_{r(k)} T_s \phi_{rq(k)} + \beta\eta T_s \phi_{rd(k)} - \beta\eta L_m T_s i_{sd(k)} \end{aligned} \quad (25)$$

Then, the following assumption is made.

A1. It is reasonable to assume that, for small values of T_s, the variation of the mechanical rotor speed over one sampling time is slower than the variation of the electrical variables such as stator currents and rotor flux. Thus, the mechanical rotor speed may be considered constant in this period, that is, $\omega_{r(k+1)} \simeq \omega_{r(k)}$.

Therefore from the discretisation of (3) and (4) and from the assumption A1, it is possible to write (24) and (25) in the following form

$$\begin{bmatrix} L_{\alpha eq(k+1)} \\ L_{\beta eq(k+1)} \end{bmatrix} = \begin{bmatrix} (1-\eta T_s) & N\omega_{r(k+1)} T_s \\ -N\omega_{r(k+1)} T_s & (1-\eta T_s) \end{bmatrix} \begin{bmatrix} L_{\alpha eq(k)} \\ L_{\beta eq(k)} \end{bmatrix} - \beta\eta L_m T_s \begin{bmatrix} \Delta i_{sq(k)} \\ \Delta i_{sd(k)} \end{bmatrix} \quad (26)$$

where $\Delta i_{sq(k)} = i_{sq(k+1)} - i_{sq(k)}$, and $\Delta i_{sd(k)} = i_{sd(k+1)} - i_{sd(k)}$ are the discrete-time variations of i_{sq} and i_{sd}, respectively.

In (26), there is the presence of rotor speed variable. Assuming that $L_{\alpha eq(k)}$ and $L_{\beta eq(k)}$ may be obtained through a low-pass filter from the discontinuous time functions $V_{\alpha(k)}$ and $V_{\beta(k)}$ as in [19], it is possible to design a discrete-time parameter observer, such as

$$\begin{aligned} \hat{x}_{(k+1)} &= (1 - KT_s)\hat{x}_{(k)} + \hat{A}T_s x_{(k)} + T_s K x_{(k)} + T_s B u_{(k)} \\ y_{(k)} &= C x_{(k)} \end{aligned} \quad (27)$$

where K is a positive gain to be chosen, A, B and C are plant parameters, and $\hat{A}$ and $\hat{x}$ are the estimates of A and x.

Assuming $L_\alpha \simeq L_{\alpha eq}$ and $L_\beta \simeq L_{\beta eq}$, which are filtered signals of the sliding mode functions $V_{\alpha(k)}$ and $V_{\beta(k)}$, it is possible to re-write (26) in the following form

$$\begin{bmatrix} L_{\alpha(k+1)} \\ L_{\beta(k+1)} \end{bmatrix} = \begin{bmatrix} (1-\eta T_s) & N\omega_{r(k+1)}T_s \\ -N\omega_{r(k+1)}T_s & (1-\eta T_s) \end{bmatrix} \begin{bmatrix} L_{\alpha(k)} \\ L_{\beta(k)} \end{bmatrix} - \beta\eta L_m T_s \begin{bmatrix} \Delta i_{sq(k)} \\ \Delta i_{sd(k)} \end{bmatrix} \quad (28)$$

Using the projected observer (27) in the system defined by (28)

$$\begin{bmatrix} \hat{L}_{\alpha(k+1)} \\ \hat{L}_{\beta(k+1)} \end{bmatrix} = (1-KT_s)\begin{bmatrix} \hat{L}_{\alpha(k)} \\ \hat{L}_{\beta(k)} \end{bmatrix} + T_s K \begin{bmatrix} L_{\alpha(k)} \\ L_{\beta(k)} \end{bmatrix} + \begin{bmatrix} -\eta T_s & N\hat{\omega}_{r(k+1)}T_s \\ -N\hat{\omega}_{r(k+1)}T_s & -\eta T_s \end{bmatrix} \begin{bmatrix} L_{\alpha(k)} \\ L_{\beta(k)} \end{bmatrix} - \beta\eta L_m T \begin{bmatrix} \Delta i_{sq(k)} \\ \Delta i_{sd(k)} \end{bmatrix} \quad (29)$$

where $\hat{L}_{\alpha(k)}$ and $\hat{L}_{\beta(k)}$ are the estimated values of $L_{\alpha(k)}$ and $L_{\beta(k)}$, respectively.

The errors of the estimative are $\overline{L}_{\alpha(k)} = \hat{L}_{\alpha(k)} - L_{\alpha(k)}$ and $\overline{L}_{\beta(k)} = \hat{L}_{\beta(k)} - L_{\beta(k)}$. These errors are given by

$$\begin{bmatrix} \overline{L}_{\alpha(k+1)} \\ \overline{L}_{\beta(k+1)} \end{bmatrix} = (1-KT_s)\begin{bmatrix} \overline{L}_{\alpha(k)} \\ \overline{L}_{\beta(k)} \end{bmatrix} + \begin{bmatrix} 0 & N\overline{\omega}_{r(k+1)}T_s \\ -N\overline{\omega}_{r(k+1)}T_s & 0 \end{bmatrix} \begin{bmatrix} L_{\alpha(k)} \\ L_{\beta(k)} \end{bmatrix} \quad (30)$$

where $\overline{\omega}_{r(k)} = \hat{\omega}_{r(k)} - \omega_{r(k)}$.

Theorem 1: Consider $\gamma \in R^+$. Then, the estimation algorithm given by

$$\overline{\omega}_{r(k+1)} = \frac{\overline{\omega}_{r(k)}}{(1+\gamma(1/2)T_s^2(L_{\alpha k}^2 + L_{\beta(k)}^2))} + \frac{\gamma((1-KT_s)T_s(-\overline{L}_{\alpha(k)}L_{\beta(k)} + \overline{L}_{\beta(k)}L_{\alpha(k)}))}{(1+\gamma(1/2)T_s^2(L_{\alpha k}^2 + L_{\beta(k)}^2))} \quad (31)$$

subject to state observer (29) and assumption A1 ensures the convergence of $\hat{\omega}_{r(k)}$ to $\omega_{r(k)}$, as $k \to \infty$.

Proof: Let a candidate Lyapunov function

$$V_k = \overline{L}_{\alpha(k)}^2 + \overline{L}_{\beta(k)}^2 + \gamma^{-1}\overline{\omega}_{r(k)}^2 \geq 0 \quad (32)$$

The difference equation ΔV_k is given by

$$\Delta V_k = V_{(k+1)} - V_{(k)} \quad (33)$$

Thus, combining (32) with (33) results

$$\Delta V_k = \overline{L}_{\alpha(k+1)}^2 - \overline{L}_{\alpha(k)}^2 + \overline{L}_{\beta(k+1)}^2 - \overline{L}_{\beta(k)}^2 + \gamma^{-1}\overline{\omega}_{r(k+1)}^2 - \gamma^{-1}\overline{\omega}_{r(k)}^2 \quad (34)$$

Defining $\Delta\overline{\omega}_{rk} \triangleq \overline{\omega}_{r(k+1)} - \overline{\omega}_{r(k)}$, it is possible to write the equality

$$\gamma^{-1}(\overline{\omega}_{r(k+1)}^2 - \overline{\omega}_{r(k)}^2) = \gamma^{-1}(2\overline{\omega}_{r(k+1)}\Delta\overline{\omega}_{rk} - \Delta\overline{\omega}_{rk}^2) \quad (35)$$

From (35) we can to re-write (34) as

$$\Delta V_k = \overline{L}_{\alpha(k+1)}^2 - \overline{L}_{\alpha(k)}^2 + \overline{L}_{\beta(k+1)}^2 - \overline{L}_{\beta(k)}^2 + 2\gamma^{-1}\overline{\omega}_{r(k+1)}\Delta\overline{\omega}_{rk} - \gamma^{-1}\Delta\overline{\omega}_{rk}^2 \quad (36)$$

Substituting (30) and (31) into (36), the result is

$$\Delta V_k = ((1-KT_s)^2 - 1)(\overline{L}_{\alpha(k)}^2 + \overline{L}_{\beta(k)}^2) - \gamma^{-1}\Delta\overline{\omega}_{rk}^2 \quad (37)$$

For any $KT_s \in (0, 1)$, then (37) results in $\Delta V_k \leq 0$, which means that the candidate Lyapunov function V_k under the estimation algorithm (31) is delaying until $\overline{L}_{\alpha(k)} = 0$ and $\overline{L}_{\beta(k)} = 0$. □

Expression (31) has the characteristic of a gradient identifier and this equation can be obtained from the Lyapunov candidate function presented in (32) aiming to cancel the terms that are not negatively defined. As a result, substituting (31) into (32) we have (37).

By the assumption A1 we have $\Delta\overline{\omega}_{rk} = \Delta\hat{\omega}_{rk}$, thus, we can calculate the estimated rotor speed $\hat{\omega}_{r(k+1)}$, such as

$$\Delta\hat{\omega}_{rk} = \hat{\omega}_{r(k+1)} - \hat{\omega}_{r(k)} = \overline{\omega}_{r(k+1)} - \overline{\omega}_{r(k)} \quad (38)$$

5 Experimental results and discussions

Experimental results were obtained in DSP-based platform using TMS320F2812, PWM voltage source inverter (VSI) and IM. The implemented system diagram is presented in Fig. 1, where an indirect field oriented control (IFOC) approach with a *qd* reference-frame rotating at synchronous speed ω_e is used. The induction machine is a Y-connected four-pole whose parameters are listed in Table 1. The switching frequency was selected at 5 kHz. The stator voltages used in the speed observer algorithm are obtained from reference voltages of the PWM VSI. The stator currents are measured by hall effect sensors. The gains of the rotor speed estimation algorithm are presented in Table 2.

Fig. 2 presents the rotor speed response and it shows the actual rotor speed (ω_r), the estimated speed ($\hat{\omega}_r$) and the speed reference (ref). We can notice the good capacity of speed estimation for the tested entire range. Fig. 3 presents the actual and estimate stator currents, it demonstrates the good stator current estimation. In Fig. 3 we can observe oscillations around the measured stator currents. These oscillations are due to the switching of the control law and can be reduced with the change in the amplitude of the sliding mode switching gain.

It is well known that the discrete-time sliding mode systems are characterised by a quasi-sliding mode band around the switching hyperplane, owing to the fact that the switching frequency is limited to the sampling rate [33]. We presented the bounds for the existence of the discrete-time switching hyperplane, such as shown in Section 3.1. These bounds vary with the estimation error, thus the variable switching gain can improve the performance of the discrete-time sliding mode observer. In this paper, the use of two

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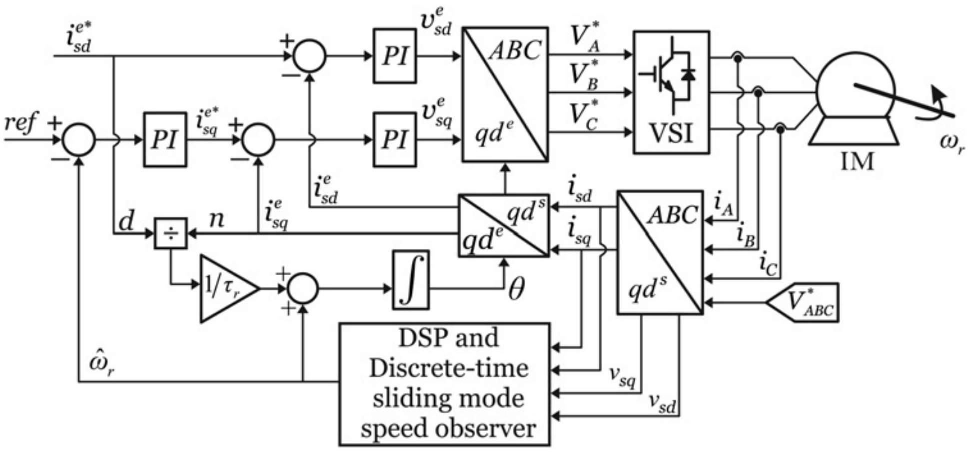

Fig. 1 *Diagram of the control scheme*

Table 1 Induction machine parameters

P, kW	n_{rated}, rpm	R_s, Ω	R_r, Ω	L_s, mH	L_r, mH	L_m, mH
1.2	1720	3.24	4.96	402.4	404.8	388.5

Table 2 Sliding mode observer gains

$V_{0\alpha}$, $V_{0\beta}$	K	γ
0.3	4000	5000

algorithms is carried out and suggested for the reduction of the quasi-sliding mode band.

5.1 Sigmoid function

The switching gain of the sliding mode function may be designed for the worst case, which may result in a relative high gain, and consequently lead to high observer activity. Here, we used a sigmoid function as the switching function in (12) and (13). The sigmoid function can improve the performance of the system owing to the reduction on the amplitude of the control law $V_{\alpha(k)}$ and $V_{\beta(k)}$ with the reduction of current estimation error. Thus, this function may be used for continuous-time systems [34] and for discrete-time systems as proposed in this study.

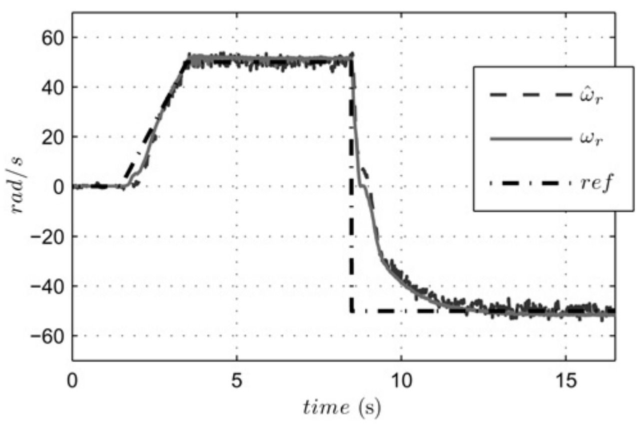

Fig. 2 *Experimental speed response*

The sigmoid function is represented as follows

$$f(x) = -K_{sig}\left(-0.5 + \frac{1}{1 + e^{-\tau_{sig}\bar{x}}}\right) \qquad (39)$$

where τ_{sig} is the time constant of the function, K_{sig} is a positive gain to be designed and $\bar{x}$ is a function of error.

5.2 Adaptive switching gain

In [35], an adaptation law is proposed which aims to find the optimal gain for the switching function. In this paper, the adaptive algorithm is implemented and compared with the results obtained with the sigmoid function. Thus, in the laws (12) and (13) the gain V_{0i} could be calculated by

$$\hat{V}_{0i(k)} = |\hat{V}_{0i(k-1)} + \lambda\, \text{sign}(s_{i(k)})\, \text{sign}(s_{i(k-1)})| \qquad (40)$$

where $\lambda > 0$ is an adaptation constant. This constant determines the coefficient of adaptation. The term $\text{sign}[s_{i(k)}]\, \text{sign}[s_{i(k-1)}]$ switches the signal of λ when the error crosses the switching surface, then the gain $\hat{V}_{0i(k)}$ decreases. When the term $\text{sign}[s_{i(k)}]\, \text{sign}[s_{i(k-1)}]$ is positive the gain $\hat{V}_{0i(k)}$ is increased forcing the sliding mode error to reach zero. Experimental results are achieved to verify the performance of the proposed rotor speed observer with the previous algorithms.

Fig. 4 presents the rotor speed response of the proposed scheme using the sigmoid function on the switching law. From Fig. 4, it is possible to verify the good convergence of estimated rotor speed to actual rotor speed, and the reduction on oscillations of estimated speed. Fig. 4 illustrates a quick speed estimation and an effective tracking

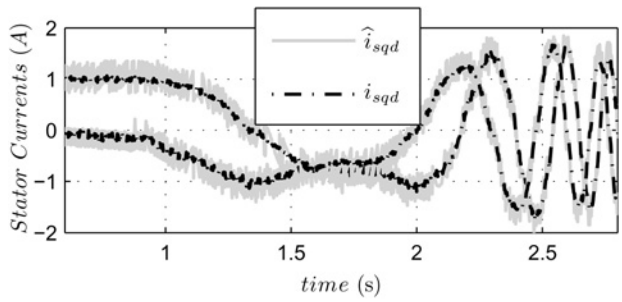

Fig. 3 *Measured stator currents*

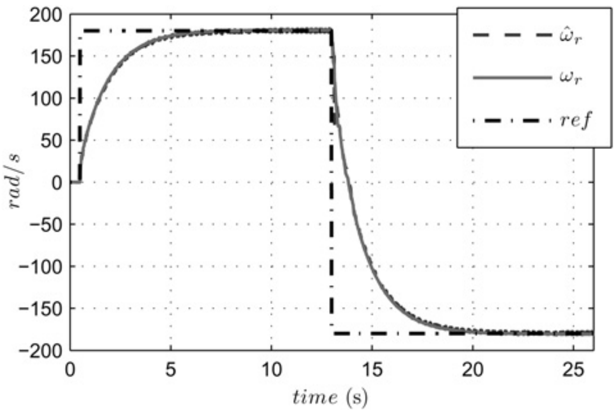

Fig. 4 *Experimental speed response using sigmoid function*

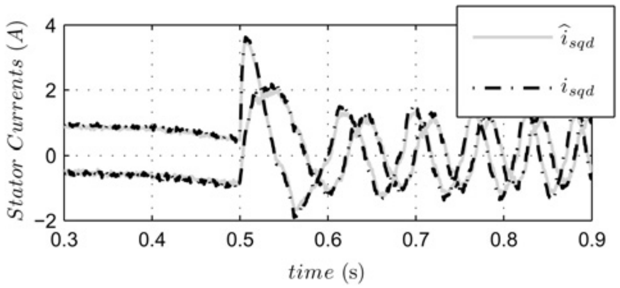

Fig. 5 *Measured and observed stator currents with sigmoid function*

performance at nominal conditions, reverse reference and zero crossing.

Fig. 5 shows the measured and the estimated stator currents. This figure evidences the reduction on oscillations of the estimated stator currents. Fig. 6 illustrates the effective estimation of L_α and L_β terms. Fig. 7 presents a second test in low and medium speeds, where Fig. 7*a* shows the rotor speed response and Fig. 7*b* demonstrates the convergence of $V_{\alpha(k)}$, which varies the amplitude with the rotor speed. The $V_{\beta(k)}$ function has similar characteristics to Fig. 7*b*.

In this study, the rotor speed range of interest is from 10% to the rated speed. The experimental results presented this range of operation; for instance, Fig. 4 presents the operation from zero to the rated speed and reversed rated speed. Fig. 7 presents the operation from 10% (about 18 rad/s) of rated speed to 54% (about 100 rad/s) of rated rotor speed.

The second algorithm implemented was the 'adaptive switching gain'. Fig. 8 illustrates the speed response of the proposed scheme using the adaptation switching gain algorithm; the good speed estimation of the scheme is demonstrated. Fig. 9 shows the measured and estimated stator currents. The oscillation reduction in the estimated stator currents is verified in comparison with Fig. 3. The

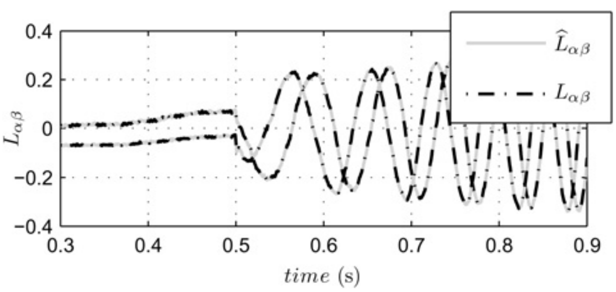

Fig. 6 *Estimation of L_α and L_β*

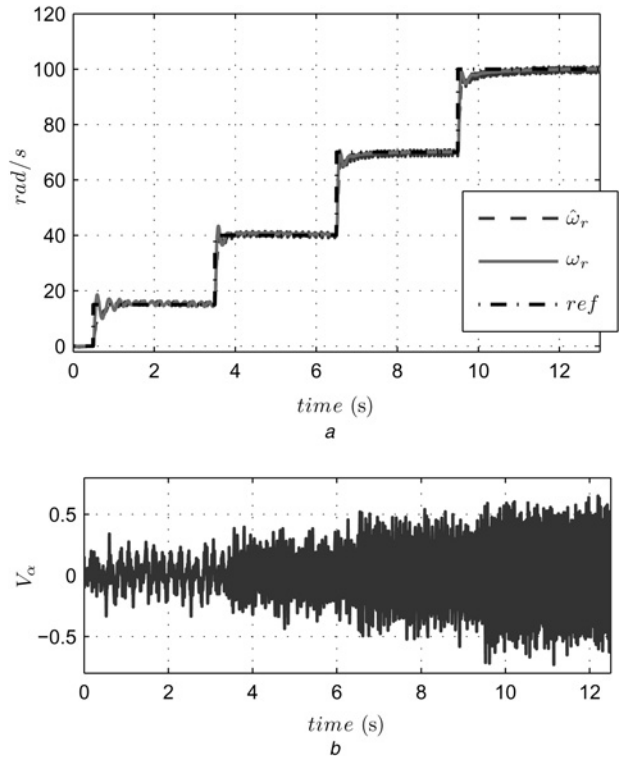

Fig. 7 *Rotor speed response using sigmoid function*

a ω_r, $\hat{\omega}_r$ and ref

b Estimation error, V_α function

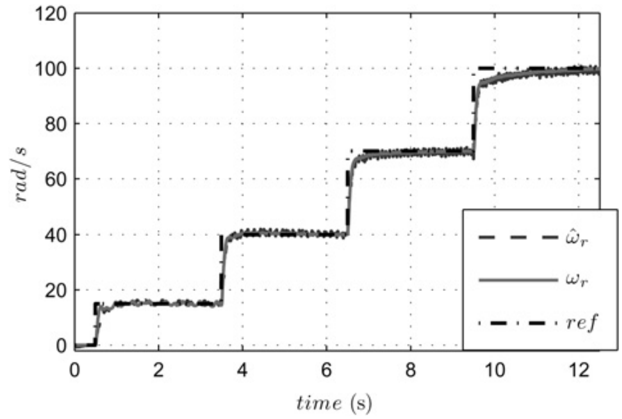

Fig. 8 *Experimental speed response with adaptive switching gain, ω_r, $\hat{\omega}_r$ and ref*

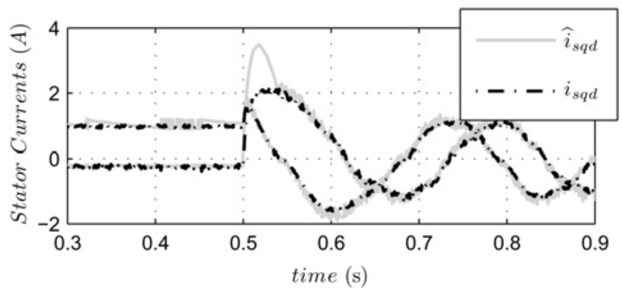

Fig. 9 *Measured and observed stator currents for scheme using the algorithm to adapte the switching gains*

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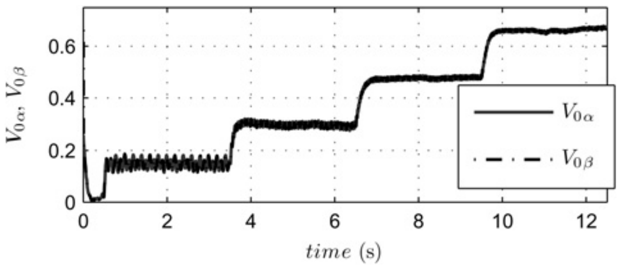

Fig. 10 *Convergence of switching gain*

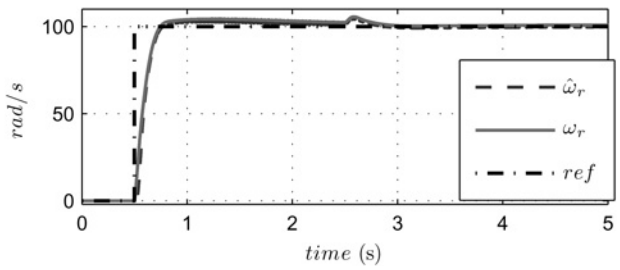

Fig. 11 *Simulation of speed response with parameter change*

results presented in Figs. 8 and 9 demonstrate that the proposed scheme has a good speed response and good speed estimation for the entire operating range. Fig. 10 presents the switching gains convergence. These gains increase with the rotor speed owing to the fact that the estimation variables $\hat{L}_\alpha$ and $\hat{L}_\beta$ increase with the rotor speed. Notice that the estimation error is large in the instant 0.5 s in Fig. 9; this is due to the low value for the switching gain in the previous instant. Thus, when the error increases the gains $V_{0\alpha}$ and $V_{0\beta}$ also increase and consequently make the estimated values track the measured values.

It is well known from the literature that the sliding mode techniques are characterised by strong robustness and, disturbance rejection, and these techniques are insensitive to parameter variations. Here, this concept is applied to develop a robust speed observer. In this study, a simulation is carried out aiming to demonstrate the robustness of the observer with respect to parameter changes. The rated parameters of simulated IM are presented in Table 1. The resistances are the parameters that have the major variation with the temperature. Thus, Fig. 11 shows the simulated speed response for a step speed reference. In this simulation at instant 2.5 s the rotor resistance is changed to 1.7 pu of the rated value, whereas the stator resistance is changed to 1.5 pu of the rated value. From Fig. 11 it is possible to verify the good speed estimation of the proposed scheme under parameter changes.

6 Conclusion

This paper has addressed a rotor speed observer for sensorless IM drives. A discrete-time sliding mode speed observer was presented and analysed. The discrete-time Lyapunov analysis demonstrates the convergence of estimated rotor speed to the actual rotor speed in the proposed system. This study presents the limits to ensure the stability of the proposed discrete-time scheme. In addition, this paper illustrates and analyses two algorithms for the reduction of quasi-sliding mode band in the sliding surface. The experimental results demonstrate that the sigmoid function and the adaptive switching gain reduce the oscillations on the estimates of stator currents and rotor speed estimation. The sliding mode observer is attractive because of the fact that this scheme is robust, has simplicity implementation and disturbance rejection. Moreover, an IFOC method was implemented combined with the scheme. Thus, the proposed method has demonstrated to be suitable for close-loop sensorless IM drives. Experimental results are used to demonstrate the good performance and the effectiveness of the technique.

7 References

1 Holtz, J.: 'Sensorless control of induction machines: with or without signal injection?', *IEEE Trans. Ind. Electron.*, 2005, **53**, (1), pp. 7–30

2 Barut, M., Bogosyan, S., Gokasan, M.: 'Speed-sensorless estimation for induction motors using extended kalman filters', *IEEE Trans. Ind. Electron.*, 2007, **54**, (1), pp. 272–280

3 Schauder, C.: 'Adaptive speed identification for vector control of induction motors without rotational transducers', *IEEE Trans. Ind. Appl.*, 1992, **28**, (5), pp. 1054–1061

4 Peng, F.-Z., Fukao, T.: 'Robust speed identification for speed-sensorless vector control of induction motors', *IEEE Trans. Ind. Appl.*, 1994, **30**, (5), pp. 1234–1240

5 Orlowska-Kowalska, T., Dybkowski, M.: 'Stator-current-based MRAS estimator for a wide range speed-sensorless induction-motor drive', *IEEE Trans. Ind. Electron.*, 2010, **57**, (4), pp. 1296–1308

6 Cardenas, R., Pena, R., Clare, J., Asher, G., Proboste, J.: 'MRAS observers for sensorless control of doubly-fed induction generators', *IEEE Trans. Power Electron.*, 2008, **23**, (3), pp. 1075–1084

7 Rashed, M., Stronach, A.: 'A stable back-EMF MRAS-based sensorless low-speed induction motor drive insensitive to stator resistance variation', *IEE Proc. Electr. Power Appl.*, 2004, **151**, (6), pp. 685–693

8 Jevremovic, V., Vasic, V., Marcetic, D., Jeftenic, B.: 'Speed-sensorless control of induction motor based on reactive power with rotor time constant identification', *IET Electr. Power Appl.*, 2010, **4**, (6), pp. 462–473

9 Vas, P.: 'Sensorless vector and direct torque control' (Oxford University Press, 1998)

10 Kim, Y.-R., Sul, S.-K., Park, M.-H.: 'Speed sensorless vector control of induction motor using extended kalman filter', *IEEE Trans. Ind. Appl.*, 1994, **30**, (5), pp. 1225–1233

11 Utkin, V.: 'Sliding mode control design principles and applications to electric drives', *IEEE Trans. Ind. Electron.*, 1993, **40**, (1), pp. 23–36

12 Lascu, C., Boldea, I., Blaabjerg, F.: 'Direct torque control of sensorless induction motor drives: a sliding-mode approach', *IEEE Trans. Ind. Appl.*, 2004, **40**, (2), pp. 582–590

13 Yan, Z., Jin, C., Utkin, V.: 'Sensorless sliding-mode control of induction motors', *IEEE Trans. Ind. Electron.*, 2000, **47**, (6), pp. 1286–1297

14 Ghafari-Kashani, A., Faiz, J., Yazdanpanah, M.: 'Integration of non-linear H∞ and sliding mode control techniques for motion control of a permanent magnet synchronous motor', *IET Electr. Power Appl.*, 2010, **4**, (4), pp. 267–280

15 Comanescu, M., Utkin, V.: 'A sliding mode adaptive MRAS speed estimator for induction motors'. IEEE Int. Symp. on Industrial Electronics, ISIE 2008, July 2008, pp. 634–638

16 Ghanes, M., Zheng, G.: 'On sensorless induction motor drives: sliding-mode observer and output feedback controller', *IEEE Trans. Ind. Electron.*, 2009, **56**, (9), pp. 3404–3413

17 Lascu, C., Boldea, I., Blaabjerg, F.: 'Comparative study of adaptive and inherently sensorless observers for variable-speed induction-motor drives', *IEEE Trans. Ind. Electron.*, 2005, **53**, (1), pp. 57–65

18 Rao, S., Buss, M., Utkin, V.: 'Simultaneous state and parameter estimation in induction motors using first- and second-order sliding modes', *IEEE Trans. Ind. Electron.*, 2009, **56**, (9), pp. 3369–3376

19 Derdiyok, A., Yan, Z., Guven, M., Utkin, V.: 'A sliding mode speed and rotor time constant observer for induction machines'. Annual Conf. on Industrial Electronics Society, 2001. IECON'01, 2001

20 Derdiyok, A.: 'Simple method for speed and rotor resistance estimation of induction machines', *IEE – Electr. Power Appl.*, 2003, **150**, (3), pp. 289–294

21 Derdiyok, A.: 'Speed-sensorless control of induction motor using a continuous control approach of sliding-mode and flux observer', *IEEE Trans. Ind. Electron.*, 2005, **52**, (4), pp. 1170–1176

22 Utkin, V., Guldner, J., Shi, J.: 'Sliding mode control in electromechanical systems' (Taylor and Francis, 1999)

23 Jung, S.-L., Tzou, Y.-Y.: 'Discrete sliding-mode control of a pwm inverter for sinusoidal output waveform synthesis with optimal sliding curve', *IEEE Trans. Power Electron.*, 1996, **11**, (4), pp. 567–577
24 Bartoszewicz, A.: 'Discrete-time quasi-sliding-mode control strategies', *IEEE Trans. Ind. Electron.*, 1998, **45**, (4), pp. 633–637
25 Gao, W., Wang, Y., Homaifa, A.: 'Discrete-time variable structure control systems', *IEEE Trans. Ind. Electron.*, 1995, **42**, (2), pp. 117–122
26 Veluvolu, K., Soh, Y.C.: 'Discrete-time sliding-mode state and unknown input estimations for nonlinear systems', *IEEE Trans. Ind. Electron.*, 2009, **56**, (9), pp. 3443–3452
27 Sarpturk, S., Istefanopulos, Y., Kaynak, O.L.: 'On the stability of discrete-time sliding mode control systems', *IEEE Trans. Autom. Control*, 1987, **32**, (10), pp. 930–932
28 Veselic, B., Perunicic-Drazenovic, B., Milosavljevic, C.: 'Improved discrete-time slidingmode position control using euler velocity estimation', *IEEE Trans. Ind. Electron.*, 2010, **57**, (11), pp. 3840–3847
29 Krause, P.C., Wasynczuk, O., Sudhoff, S.D.: 'Analysis of electric machinery' (IEEE Press, Piscataway, NJ, 1995, 2nd edn.)
30 Sira-Ramirez, H.: 'Non-linear discrete variable structure systems in quasi-sliding mode', *Int. J. Control*, 1991, **54**, (5), pp. 1171–1187
31 Milosavljevic, C.: 'General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems', *Autom. Remote Control*, 1985, **46**, pp. 307–314
32 Sabanovic, A., Fridman, L.M., Spurgeon, S.: 'Variable structure systems from the principles to implementation' (The Institution of Electrical Engineers, 2004, 1st edn.)
33 Utkin, V.I., Guldner, J., Shi, J.: 'Sliding mode control in electromechanical systems' (Taylor & Francis, 1999, 1st edn.)
34 Kim, H., Son, J., Lee, J.: 'A high-speed sliding-mode observer for the sensorless speed control of a pmsm', *IEEE Trans. Ind. Electron.*, 2011, **58**, (9), pp. 4069–4077
35 Monsees, G., Scherpen, J.: 'Adaptive switching gain for a discrete-time sliding mode controller'. Proc. American Control Conf., 2000, vol. 3, pp. 1639–1643

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