

Lista 1 - Determinantes

(1)

$$\textcircled{1} a = \begin{vmatrix} -3 & 2 \\ -5 & 1 \end{vmatrix} \quad a = -3 + 10 \\ a = 7$$

$$b = \begin{vmatrix} 2 & 6 \\ 4 & 10 \end{vmatrix} \quad b = 20 - 24 \\ b = -4$$

$$3a + b^2 = 3(7) + (-4)^2 = 21 + 16 = \textcircled{37}$$

$$\textcircled{2} A - x \cdot B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} - x \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2-4x & 1-2x \\ 3-3x & 4+x \end{pmatrix}$$

$$\det \begin{pmatrix} 2-4x & 1-2x \\ 3-3x & 4+x \end{pmatrix} = 0$$

$$\begin{vmatrix} 2-4x & 1-2x \\ 3-3x & 4+x \end{vmatrix} = 0$$

$$8 + 2x - 16x - 4x^2 - (3 - 6x - 3x + 6x^2) = 0 \\ -4x^2 - 6x^2 - 14x + 9x + 8 - 3 = 0 \\ -10x^2 - 5x + 5 = 0 \quad \div -5$$

$$2x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2 \cdot 2}$$

$$x = \frac{-1 \pm 3}{4} \begin{cases} \frac{-4}{4} = -1 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

$$S = \left\{ -1, \frac{1}{2} \right\}$$

$$\textcircled{3} \text{ a) } \cancel{10} + 2 + 12x - 12 - 2x - \cancel{10} = 0$$

$$10x = 10$$

$$x = 1$$

$$S = \{1\}$$

$$\text{b) } (x+1)(x-1) \cdot x + 6x + 3x - x^3 - 9(x-1) - 2(x+1) = 0$$

$$(x^2-1) \cdot x + 9x - x^3 - 9x + 9 - 2x - 2 = 0$$

$$\cancel{x^3} - x - \cancel{x^3} + 7 - 2x = 0$$

$$-3x = -7$$

$$x = \frac{7}{3}$$

$$S = \left\{ \frac{7}{3} \right\}$$

$$\textcircled{4} \begin{vmatrix} 0 & 0 & 1 \\ 2 & x & 4 \\ 1 & 1 & y \end{vmatrix} = 1$$

$$2 - x = 1$$

$$-x = 1 - 2$$

$$\boxed{x = 1}$$

$$\begin{vmatrix} 1 & x & 0 \\ 0 & y & 1 \\ 1 & 0 & 1 \end{vmatrix} = 8$$

$$y + x = 8$$

$$y + 1 = 8$$

$$\boxed{y = 7}$$

$$\textcircled{5} N = 50 + \det(A \cdot B)$$

$$A \cdot B = \begin{pmatrix} 2 & -1 \\ -2 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2-2 & 4-1 & 6-1 \\ 2+4 & -4+2 & -6+2 \\ 2 & 1 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} -4 & 3 & 5 \\ 6 & -2 & -4 \\ 2 & 1 & 1 \end{pmatrix}$$

$$\det A \cdot B = 30 - 24 + 20 - 18 = 0$$

$$\boxed{N = 50}$$

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$$\textcircled{6} B = C \begin{pmatrix} x+y & x+z \\ z-y & z-x \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 2 & 4 \end{pmatrix}$$

$$x+y = 4 \rightarrow x = 4-y \quad \textcircled{1}$$

$$z-y = 2 \rightarrow z = 2+y \quad \textcircled{2}$$

$$x+z = 6$$

$$z-x = 4$$

$$\hookrightarrow 2+y-x = 4$$

$$-x = 4-2-y$$

$$x = y-2 \quad \textcircled{3}$$

$$\textcircled{1} \text{ e } \textcircled{3} \begin{cases} 4-y = y-2 \\ -y-y = -2-4 \\ -2y = -6 \end{cases} \begin{matrix} x = 4-y \\ x = 4-3 \\ x = 1 \end{matrix} \quad \textcircled{1}$$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 5 & 3 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

$$\begin{matrix} y = 3 \\ z = 2+y \\ z = 2+3 \\ z = 5 \end{matrix} \quad \textcircled{2}$$

$$\det A = 3 + 125 + 9 - 45 - 5 - 15$$

$$\boxed{\det A = 72}$$

$$\textcircled{7} A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad a_{ij} = j^{-i^2}$$

$$A = \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}$$

$$\boxed{\det A = 3}$$

$$\textcircled{8} \quad A \cdot B = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \cdot (-2 \mid 3 \mid 5) \quad \textcircled{4}$$

$$A \cdot B = \begin{pmatrix} 2 & -3 & -5 \\ -4 & 6 & 10 \\ 6 & -9 & -15 \end{pmatrix}$$

$$(A \cdot B) \cdot C = \begin{pmatrix} 2 & -3 & -5 \\ -4 & 6 & 10 \\ 6 & -9 & -15 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ -2 & 1 & 2 \\ 3 & -1 & 4 \end{pmatrix}$$

$$(A \cdot B) \cdot C = \begin{pmatrix} -2+6-15 & -3+5 & -6-20 \\ 4-12+30 & 6-10 & 12+40 \\ -6+18-45 & -9+15 & -18-60 \end{pmatrix}$$

$$\det (A \cdot B) \cdot C = \begin{vmatrix} -11 & 2 & -26 \\ 22 & -4 & 52 \\ 33 & 6 & -68 \end{vmatrix}$$

$$\det = -29/92 - 34/2 + 34/2 - 34/32 + 29/92 + 34/2$$

$$\boxed{\det = 0}$$