Lista 2 - Matrizes

① $X + A - (B + C) = 0$

$X = B + C - A$

$X = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad X = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

② $(X + A)^t = B$

$X + A = B^t$

$X = B^t - A$

$X = \begin{pmatrix} 5 & -2 \\ 1 & 0 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 4 & 3 \end{pmatrix}$ $\quad X = \begin{pmatrix} 4 & -4 \\ 2 & 0 \\ -1 & -1 \end{pmatrix}$

③ $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $\quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix}$

a) $A - B \Rightarrow \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

b) $B - A \Rightarrow \begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

c) $(A + B)^t \Rightarrow A + B = \begin{pmatrix} 3 & -3 \\ 7 & 1 \end{pmatrix}$ $\quad (A+B)^t = \begin{pmatrix} 3 & 7 \\ -3 & 1 \end{pmatrix}$

d) $A^t - B^t = \begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

4) $2x - A + \frac{1}{2}B = 0$ $\quad mmc = 2$

$4x - 2A + B = 0$

$X = \frac{2A - B}{4} = \frac{1}{4}(2A - B)$

$X = \frac{1}{4}\left(2.\begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}10\\4\\-8\end{pmatrix}\right)$

$X = 1/4\left[\begin{pmatrix}6\\4\\-2\end{pmatrix} + \begin{pmatrix}-10\\-4\\+8\end{pmatrix}\right]$

$X = \frac{1}{4}\begin{pmatrix}-4\\0\\6\end{pmatrix}$

$X = \begin{pmatrix}-1\\0\\3/2\end{pmatrix}$

5) $\begin{cases} 2x - y = A \\ 3x + 2y = B \end{cases}$

$y = 2x - A$

$3x + 2(2x - A) = B$

$3x + 4x - 2A = B$

$7x = B + 2A$

$\boxed{x = \frac{B + 2A}{7}}$

e $y = 2x - A$

$y = 2\left(\frac{B+2A}{7}\right) - A$

$y = \frac{2B + 4A}{7} - \frac{7A}{7}$

$\boxed{y = \frac{2B - 3A}{7}}$

$$X = \frac{1}{7}\left[\begin{pmatrix} 4 & 2 \\ -1 & 0 \end{pmatrix} + 2\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\right]$$

$$X = \frac{1}{7}\begin{pmatrix} 6 & -2 \\ -1 & 2 \end{pmatrix} \qquad X = \begin{pmatrix} 6/7 & -2/7 \\ -1/7 & 2/7 \end{pmatrix}$$

$$Y = \frac{1}{7}\left[2\begin{pmatrix} 4 & 2 \\ -1 & 0 \end{pmatrix} - 3\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\right]$$

$$Y = \frac{1}{7}\left[\begin{pmatrix} 8 & 4 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} 3 & -6 \\ 0 & 3 \end{pmatrix}\right]$$

$$Y = \frac{1}{7}\begin{pmatrix} 5 & 10 \\ -2 & -3 \end{pmatrix} \qquad Y = \begin{pmatrix} 5/7 & 10/7 \\ -2/7 & -3/7 \end{pmatrix}$$

⑥ $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad A = \begin{pmatrix} 7 & 11 \\ 10 & 14 \end{pmatrix}$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \qquad B = \begin{pmatrix} -7 & -10 \\ -11 & -14 \end{pmatrix}$$

$$C = A + B$$

$$C = \begin{pmatrix} 7 & 11 \\ 10 & 14 \end{pmatrix} + \begin{pmatrix} -7 & -10 \\ -11 & -14 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$C^2 = C \cdot C \Rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(7) $A_{3\times3} \times X_{3\times1} = C_{3\times1}$ $\quad X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 2 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} a+0+0 \\ 2a+b+0 \\ a+3b+2c \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 11 \end{pmatrix}$$

$2a + b = 8$
$6 + b = 8$
$b = 2$

$a = 3$

$$X = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$a + 3b + 2c = 11$
$3 + 6 + 2c = 11$
$2c = 11 - 9$
$C = 1$

(8) $\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 9-1 & -3-2 \\ 3+2 & -1+4 \end{pmatrix}$ $\quad A^2 = A \cdot A$

$$\Downarrow$$

$$\begin{pmatrix} 8 & -5 \\ 5 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 8 & -5 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 8x - 5y \\ 5x + 3y \end{pmatrix} = \begin{pmatrix} 13 \\ 2 \end{pmatrix}$$

$$\begin{cases} 8x - 5y = 13 \\ 5x + 3y = 2 \end{cases}$$

$8x = 13 + 5y \quad x = \dfrac{13 + 5y}{8}$

$5\left(\dfrac{13 + 5y}{8}\right) + 3y = 2$

$\dfrac{65 + 25y + 24y}{8} = \dfrac{16}{8}$

→b

$49y = 16 - 65$

$y = \frac{-49}{49}$

$\boxed{y = -1}$

$x = \frac{13 + 5(-1)}{8}$

$x = \frac{8}{8}$

$\boxed{x = 1}$

(9) $A^2 = A \cdot A \Rightarrow \begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix} = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$

$$\begin{pmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{pmatrix} = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$$

$1 + 2a = 9$

$2a = 8$

$\boxed{a = 4}$

$2 + 2b = -4$

$2b = -6$

$\boxed{b = -3}$

(10) Não tem

(11) Não é para fazer

(12) a) $X = 3\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} - 2\left[\begin{pmatrix} -1 & -3 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}\right]$

$X = \begin{pmatrix} 9 & 15 \\ -6 & 12 \end{pmatrix} - 2\begin{pmatrix} 2 & 8 \\ 4 & 11 \end{pmatrix}$

$X = \begin{pmatrix} 5 & 11 \\ -14 & -10 \end{pmatrix}$

(6)

b) $X = B - C - 3C$

$X = B - 4C$

$$X = \begin{pmatrix} -1 & -3 \\ 6 & 7 \end{pmatrix} - 4\begin{pmatrix} 0 & -4 \\ 1 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 13 \\ 2 & 11 \end{pmatrix}$$

c) $X = A \cdot B - C$

$$X = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & -3 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 1 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} -3+30 & -9+35 \\ 2+24 & 6+28 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 1 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 27 & 30 \\ 25 & 35 \end{pmatrix}$$

d) $X = A^2 = A \cdot A$

$$X = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$$

$$X = \begin{pmatrix} 9-10 & 15+20 \\ -6-8 & -10+16 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 35 \\ -14 & 6 \end{pmatrix}$$

e) $X = B \cdot D^t$

$$X = \begin{pmatrix} -1 & -3 \\ 6 & 7 \end{pmatrix} \cdot \begin{pmatrix} -4 & 5 & -6 \\ 1 & -3 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 4-3 & -5+9 & 6+0 \\ -24+7 & 30-21 & -36+0 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 4 & 6 \\ -17 & 9 & -36 \end{pmatrix}$$

f) $\frac{1}{2}D - \frac{2}{3}D = \frac{3D - 4D}{6} = -\frac{1}{6}D$

$$X = -\frac{1}{6}\begin{pmatrix} -4 & 1 \\ 5 & -3 \\ -6 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 2/3 & -1/6 \\ -5/6 & 1/2 \\ +1 & 0 \end{pmatrix}$$

"Qualquer resposta diferente, enviem a dúvida antes de apagar, pois o erro poderá estar na resolução acima apresentada."